Essential Mathematics & Statistics for Science: Second Edition by Graham Currell & Antony Dowman (John Wiley & Sons)

Answers to 'Q' Questions

10 t-Tests and F-tests

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Q10.1

i) H_O: Level of pollutant is NOT greater than 3.50×10^{-8} g L^{-1}, $\mu \leq 3.50 \times 10^{-8}$ g L^{-1}

H_1: Level of pollutant IS greater than 3.50×10^{-8} g L^{-1}, $\mu > 3.50 \times 10^{-8}$ g L^{-1}

ii) Confidence Level = 95%

Thus, Significance Level, $\alpha = 0.05$

iii) Mean, $\bar{x} = 3.585 \times 10^{-8}$ g L^{-1}

Standard Deviation, s = 0.080×10^{-8} g L^{-1}

iv) This is a One-Sample *t*-Test, and the *t*-statistic is calculated using [10.1]:

$$t_{STAT} = \frac{(\bar{x} - \mu_O)}{s/\sqrt{n}} = \frac{(3.585 - 3.50)}{0.080/\sqrt{6}} = 2.60$$

Note that, since all data values have the same multiplier, $\times 10^{-8}$ g L^{-1}, this multiplier will cancel out in the above calculation. It can therefore be omitted in the calculation.

v) There is a specific sense of direction in H_1, hence:

Tails = 1

vi) Degrees of Freedom, df = n - 1 = 5

vii) From tables:

$t_{CRIT} = 2.02$

viii) $t_{STAT} > +t_{CRIT}$

Hence we can Accept H_1 at a significance level of 0.05.

There is sufficient evidence to state at 95% confidence that the pollution level does exceed 3.50×10^{-8} g L^{-1}.

Q10.2

	Set	t_{STAT}	t_{CRIT}	*p*-value	Decision	Conclusion
	X	-2.05	2.13		$t_{STAT} > -t_{CRIT}$	Insufficient evidence to accept H_1 at 95%

						level
	Y			0.048	$p < 0.05$	Accept H_1 at 95% level

Q10.3

i)	Scientist 1 calculates the 95% Confidence Interval correctly as 8.69 ± 0.42. The Confidence interval is 8.27 to 9.11. As 8.34 lies within this range, the scientist would NOT claim that the pH was not equal to 8.34.
ii)	As 8.34 is within the range, the p value will be > 0.05. So only c) 0.088 is possible.
iii)	[9.1] *p*-value for a 2-*tailed* test= 2 × *p*-value for a 1-*tailed* test. So from ii) *p*-value for a 1-*tailed* test = 0.088/2 = 0.044 Again this is c).

Q10.4

The Hypotheses for this problem are:

H_O: Mean diameter of the two powders are not different: $\mu_A = \mu_B$
H_1: Mean diameter of the two powders are different: $\mu_A \neq \mu_B$

This is a Two-Sample, 2-tailed, test

Significance Level, $\alpha = 0.05$

	A	B
Means, $\bar{x}_A$ and $\bar{x}_B$	6.65	4.28
Standard Deviations, s_A and s_B	3.91	2.83
Sample sizes, n_A and n_B	11	16

This is a Two-Sample t-Test: using [10.2]
Pooled Standard Deviation, $s' = \sqrt{[\{(n_A - 1)s_A^2 + (n_B - 1)s_B^2\}/(n_A + n_B - 2)]}$

$s' = \sqrt{[\{(11 - 1)3.91^2 + (16 - 1)2.83^2\}/(11 + 16 - 2)]} = 3.31$

The t-statistic is calculated using [10.3]:

$$t_{STAT} = \frac{(\bar{x}_A - \bar{x}_B)}{s' \times \sqrt{\{1/n_A + 1/n_B\}}} = \frac{(6.65 - 4.28)}{3.31 \times \sqrt{\{1/11 + 1/16\}}} = 1.83$$

Degrees of Freedom, $df = n_A + n_B - 2 = 11 + 16 - 2 = 25$

Critical t-value for
df = 25, α = 0.05, Tails = 2 is t_{CRIT} = 2.06

As $t_{STAT} < + t_{CRIT}$ and $t_{STAT} > - t_{CRIT}$

we Do Not Reject the Null Hypothesis at a confidence level of 95%.

We do not have sufficient evidence to claim that the two powders do come from original sources with significantly different mean particle diameters.

Q10.5

The paired t-test is more powerful as it has produced a much lower p value and has enabled us to Accept the Proposed Hypothesis.

Q10.6

We apply a paired t-Test to the data set differences:{15, -3, 12, 6, 21, -2, 10}

The Hypotheses for this problem are:

H_O: The true difference score is NOT different from zero, $\mu_D = \mu_O$.
H_1: The true difference score IS different from zero, $\mu_D \neq \mu_O$.

where the value for comparison, $\mu_O = 0$

Significance Level, α = 0.05
Mean value of sample = 8.43

Standard Deviation of sample = 8.77

Sample size = 7

We calculate the t-value for the One Sample test, with $\mu_O = 0$

$$t_{STAT} = \frac{(\bar{x} - \mu_O)}{s/\sqrt{n}} = \frac{(8.43 - 0)}{8.77/\sqrt{5}} = 2.54$$

Look up t_{CRIT} for 2-tails, α = 0.05 and df = 7-1 = 6 which gives:

t_{CRIT} = 2.45

We can conclude that, because

$t_{STAT} > t_{CRIT}$

we can accept the Proposed Hypothesis H_1

Q10.7

i) $F_{(1,0.05,9,7)} = 3.68$

ii) $F_{(2,0.05,15,9)} = 3.77$

$s_B > s_A$ so find $F = \frac{s_B^2}{s_A^2} = \frac{75.5^2}{37.8^2} = 3.989 = F_{STAT}$

F_{CRIT} , the 2-tailed critical value for (6-1), (7-1) degrees of freedom at 0.05 = 5.99

As $F_{STAT} < F_{CRIT}$ we do not Reject H_0

Q10.8

i) 2-tailed test

$$F_{STAT} = \frac{s_Q^2}{s_P^2} = \frac{195.5}{85.0} = 2.3$$

F_{CRIT} , the 2-tailed critical value for (21-1), (21-1) degrees of freedom at 0.05 = 2.46

As $F_{STAT} < F_{CRIT}$ we do not Reject H_0

ii) Proposed hypothesis, variance P < variance Q This is a 1-*tailed* test

$$F_{STAT} = \frac{s_Q^2}{s_P^2} = \frac{195.5}{85.0} = 2.3$$

F_{CRIT} , the 1-*tailed* critical value for (21-1), (21-1) degrees of freedom at 0.05 = 2.12

As $F_{STAT} > F_{CRIT}$ we accept $H_{1,}$ the variances are different

Q10.9

i) No, the question asks to test for a difference in variances, not that one is greater than another.

ii) The variance of Y > variance X so we place Y in the numerator

$$F_{STAT} = \frac{s_Y^2}{s_X^2} = \frac{10.47}{2.47} = 4.24$$

F_{CRIT} , the 2-tailed critical value for (7-1), (8-1) degrees of freedom at 0.05 = 5.12

As $F_{STAT} < F_{CRIT}$ we do not Reject H_0

iii) This requires a 2 sample 2-*tailed* *t*-test

A test using Excel returns a *p*-value of 0.0418 so we can conclude at 95%

level the means are different.