Essential Mathematics & Statistics for Science: Second Edition by Graham Currell & Antony Dowman (John Wiley & Sons)

Answers to 'Q' Questions

6 Rates of Change

To navigate, use the Bookmarks in the PDF file

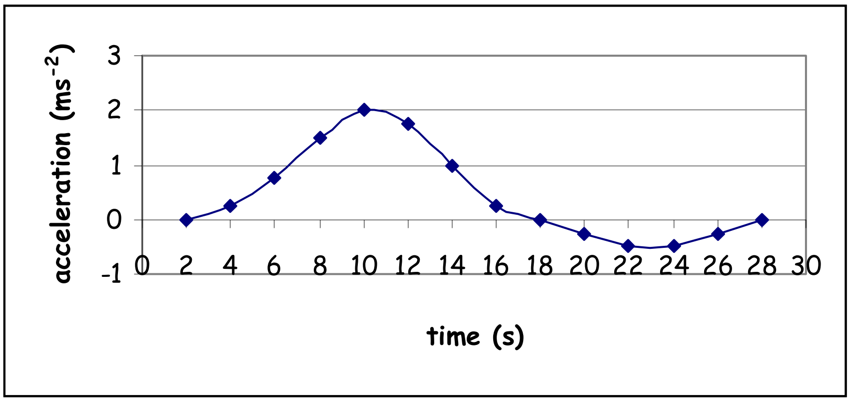
Q6.1

Calculate the missing values in the Table below:

Initial distance (m)	Final distance (m)	Initial time (s)	Final time (s)	Speed (m s^{-1})
2000	4000	100	150	40
2000	**5000**	100	200	30
2000	6000	**300**	400	40
2000	**-2000**	100	200	- 40
5000	3000	0	200	**-10**

Q6.2

Time (s)	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
Speed (ms^{-1})	20	20	20	21	23	27	31	34	35	35	35	34	33	32	32	32
Acceleration (ms^{-2})		0	0.25	0.75	1.5	2	1.75	1	0.25	0	-0.25	-0.5	-0.5	-0.25	0	



Q6.3

i) Using the triangle from time 20 to 60 minutes, slope = $(37.85\times10^{4})/(60-20)$

	= 9460. This is nearly the same as the actual slope 9350 given in the book answer to Example 6.6.
ii)	The tangent at 90 minutes crosses the x-axis at about 50 minutes. From the triangle from 50 to 100 minutes the slope is ~ $98\times10^4/(100-50)$ = 1.96×10^4 about twice the rate at 60 minutes

Q6.4

$$\frac{dN_t}{dt} = kN_t$$

Compare this with Example 6.8, the solution will be:

$N_t = N_0 e^{kt}$.

Note that this is the standard growth equation, *k* will be a positive value.

Q6.5

The decay of a bacterial population is described by the equation:

$N_t = 5.6\times10^8 \times \exp(-0.4\times t)$

where N_t is the population at a time *t* (s). Differentiating:

$$\frac{dN_t}{dt} = 5.6\times10^8\times(-0.4)\times\exp(-0.4\times t) = -2.24\times10^8\times\exp(-0.4\times t)$$

i) $t = 0$ s. $\left(\frac{dN_t}{dt}\right)_{t=0} = -2.24\times10^8\times\exp(-0.4\times0)\ \text{s}^{-1} = -2.24\times10^8\ \text{s}^{-1}$

The'–'sign means the number is reducing by 2.24×10^8 per second

ii) $t = 5$ s. $\left(\frac{dN_t}{dt}\right)_{t=5} = -2.24\times10^8\times\exp(-0.4\times5) = -3.03\times10^7\ \text{s}^{-1}$

Note the differential is negative as the number is decreasing.

iii) $t = 10$ s. $\left(\frac{dN_t}{dt}\right)_{t=10} = -2.24\times10^8\times\exp(-0.4\times10) = -4.10\times10^6\ \text{s}^{-1}$

Note the differential is negative as the number is decreasing.

Q6.6

The volumes, *V* (m^3), of adult animals of a particular species are related approximately to their heights, *h* (m), by the equation:

$V = 0.04\times h^3$

i) $\frac{dV}{dh} = 3\times0.04\ h^2 = 0.12\ h^2$

ii) Hence estimate the increase in volume (ΔV) of an animal of height, *h* = 1.6 m, if it then grows by (Δh =) 0.02 m.

a) $\Delta V \approx \left(\frac{dV}{dh}\right)_{h=1.6}\times\Delta h = 0.12\times1.6^2\times0.02 = 6.14\times10^{-3}\ \text{m}^3$

b) Alternatively we could find the slope at *h* = 1.61 m which is the average slope between 1.60 and the final 1.62 m. In this case:

$$\Delta V \approx \left(\frac{dV}{dh}\right)_{h=1.61} \times \Delta h = 0.12\times 1.61^2\times 0.02 = 6.22\times 10^{-3}\ \text{m}^3$$

Note that these 2 methods give very similar answers.