Essential Mathematics & Statistics for Science: Second Edition by Graham Currell & Antony Dowman (John Wiley & Sons)

Answers to 'Q' Questions

4 Linear Relationships

To navigate, use the Bookmarks in the PDF file

Q4.1

L is the determinate variable and *G* the indeterminate variable.
Hence *G* should be plotted against *L* (ie *L* on the x-axis)

i) True

ii) False

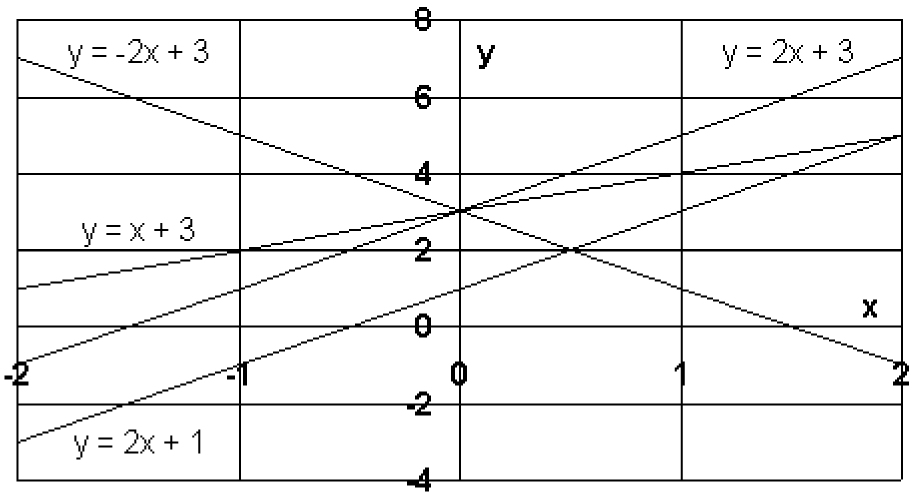
iii) False

Q4.2

i) The co-ordinates (x,y) of points are:

x	-2	-1	0	+1	+2
y	-1	1	3	5	7

ii) See line $y = 2x + 3$ plotted on graph below:



iii) $y = 3$
This is called the 'intercept'.

iv) No: for $x = 1.5$, $y = 2 \times 1.5 + 3 = 3 + 3 = 6$

v) Yes: $x = 0.5$, $y = 2 \times 0.5 + 3 = 1 + 3 = 4$ (see graph in ii))

vi) No: for $x = -0.5$, $y = 2 \times -0.5 + 3 = -1 + 3 = 2$

vii) $y = - + 3$: Note this line has a slope of –2 but still has an intercept of 3

viii) $y = 2x + 1$ same slope as first line but the intercept is now 1. The 2 lines are parallel

ix) $y = x + 3$ a slope of 1 and an intercept of 3

Q4.3

i) Substituting for α

$L = \{1 + 0.000019 \times T\} \times 1.2100$

Multiplying out the bracket

$L = 1.2100 + (2.299 \times 10^{-5}) \times T = (2.299 \times 10^{-5}) \times T + 1.2100$

ii) Slope, $m = 2.299\times10^{-5}$ m $^{o}C^{-1}$

Intercept, $c = 1.2100$ m

Q4.4

Rearrange the equation into the form: $y = mx + c$

$4y = -3x - 2$ then

$y = -(3/4)x - (2/4) = -0.75x – 0.5$

Compare this equation with $y = mx + c$ to derive:

Slope, ***m* = -0.75** and Intercept, ***c* = -0.5**

Q4.5

We can substitute the slope, $m = +4$, and the intercept, $c = -3$, into the straight-line equation to obtain the equation of the line:

$y = mx + c = 4x - 3$.

The value of x when $y = 4$ is then found by substituting $y = 4$ into the equation:

$4 = 4x - 3$

Rearranging the equation gives

$\mathbf{x = (4+3)/4 = 7/4}$

Q4.6

i) $T = 40 \times W + 20$, which could also be written as:

$T = 40W + 20$

ii) For a time of 2 hours, $T = 120$ minutes, giving the equation for the heaviest joint of meat, W:

$120 = 40W + 20$

$W = (120 - 20) / 40 = 2.5$kg

Q4.7

i) When $t = 0$ $s = 140$, hence the constant, 'c', must be +140 not -140.
As t increases the car gets closer to the junction, hence s must decrease.
Hence the 'slope, m' in the equation must be negative.
The correct equation must be:
(c) $s = -20t + 140$

ii) At the junction, $s = 0$, hence
$0 = -20t + 140$, and thus
$t = 140 / 20 = 7$ s

iii) Substituting directly into the equation:
$s = -20 \times 20 + 140 = -400 + 140 = -260$
The negative value of -260 means that the car will have gone 260m **past** the junction.

Q4.8

i) Slope, $m = (3 - 1) / (2 - 1) = 2/1 = 2$

ii) Slope, $m = (3 - 1) / (2 - (-1)) = 2/3$

iii) Slope, $m = (-3 - 1) / (2 - 1) = -4/1 = -4$

iv) Slope, $m = (3.5 - 3.0) / (1 - 2) = 0.5 / (-1) = -0.5$

v) Slope, $m = (1- 0) / (0 -1) = 1 / (-1) = - 1$

vi) Slope, m = $(-3.5 - 3.0) / (1.0 - 2.0) = -6.5 / (-1) = +6.5$

Q4.9

i) Substitute the point (0,3) into equation: $y = 2.0x + c$
$3 = 2.0 \times 0 + c$
$3 = 0 + c$
Intercept, $c = 3$
The equation is then written as: $y = 2.0x + 3$

ii) Substitute the point (2,3) into equation: $y = 0.5x + c$
$3 = 0.5 \times 2 + c$
$3 = 1 + c$
Intercept, $c = 2$
The equation is then written as: $y = 0.5x + 2$

iii) Substitute the point (2,3) into equation: $y = -0.5x + c$

$3 = -0.5 \times 2 + c$

$3 = -1 + c$

Intercept, $c = 4$

The equation is then written as: $y = -0.5x + 4$

Q4.10

The first step is to convert the data in the question into co-ordinates in a *C*-*F* graph:

(100, 212) and (0, 32).

Slope of graph $m = (212-32)/(100-0) = 180/100 = 9/5$.

Intercept c is on the $y(F)$-axis occurs at the point (0,32), ie $c = 32$.

The equation is therefore:

$\boldsymbol{F = (9/5)C + 32}$

When $F = 0$, we can substitute the value into the equation:

$0 = (9/5)C + 32$

which on rearrangement gives

$C = -32 / (9/5) = -32\times(5/9) =$ **-17.78.**

Q4.11

The slope of the line $y = 3x + 2$, is $m = 3$.

A parallel line has the same slope, $m = 3$. We can calculate the value of '*c*' for the new line by substituting the values of the point (1,1) into the equation:

$1 = 3 \times 1 + c$

giving the value of $c = -2$. The equation is:

$\boldsymbol{y = 3x - 2}$

Q4.12

The slope of the line $y = 3x + 2$, is $m = 3$.

A perpendicular line has a slope, m', such that $m \times m' = -1$, giving $m' = -1/3$

We can calculate the value of '*c*' for the new line by substituting the values of the point (1,1) into the equation:

$1 = (-1/3) \times 1 + c$

giving the value of $c = 1 + 1/3 = 4/3$. The equation becomes:

$\boldsymbol{y = (-1/3) \times x + 4/3}$,

Multiplying both sides by 3 gives

$\boldsymbol{3y = 4 - x}$

Q4.13

A vertical line has a constant value of '*x*' for every possible value of '*y*'. The equation is therefore:

$\boldsymbol{x = -1.5}$

Q4.14

Slope, m, is given by $m = (4-3)/(4-2) = 1/2$, giving
$y = 0.5x + c$
Substituting the values for one point into the equation:
$4 = 0.5 \times 4 + c = 2 + c$
which gives $c = 2$, and the equation of the line:
$\boldsymbol{y = 0.5x + 2}$.

i) Substitute $x = 3.8$
$y = 0.5 \times 3.8 + 2 = 1.9 + 2 = 3.9$

ii) Substitute $y = 5.6$
$5.6 = 0.5 \times x + 2$
$3.6 = 0.5 \times x$ *so* $x = 7.2$

iii) The value of the intercept, $c = 2$, gives the point at which the line passes through the y-axis: **(0,2).**

iv) $y = 0$ when the line crosses the x axis
$0 = 0.5 \times x + 2$
$0.5 \times x = -2$
$x = -4$

Q4.15

The first step is to convert the data in the question into co-ordinates in a *t*-*h* graph:
(0,15) and (20, 20).
Slope of graph $m = (20-15)/(20-0) = 5/20 = 0.25$.
Intercept c is on the y(h)-axis occurs at the point (0,15), i.e. $c = 15$.
The equation is therefore:
$\boldsymbol{h = 0.25t + 15}$

i) When $t = 5$, we can substitute the value into the equation:
$h = 0.25 \times 5 + 15 =$ **16.25cm**
This is interpolation

ii) When $t = 20 + 8$, we can substitute the value into the equation:
$h = 0.25 \times 28 + 15 =$ **22cm**
This is extrapolation

Q4.16

i) The ground rises 50 m = (250 – 200) m in a horizontal distance of 400 m. Thus the slope, m = 50/400 = 0.125

ii) Angle = $\tan^{-1}(0.125)$ = 0.124 radians *or* 7.125^{o}

Q4.17

The step-up rate is the variable that is 'independent' and can be chosen by the experimenter.
Hence, the step-up rate is the independent variable that should be placed on the *x*-axis - see 4.1.2.
The heart-rate is an 'outcome' that is 'dependent' on processes of the experiment and the values of the independent *x*-variable.
Hence, the heart-rate is the dependent variable and should be placed on the y-axis.

There will be very little uncertainty in the values for the step-up rate on the *x*-axis.
The major uncertainty will be in the heart-rate on the *y*-axis.
The balance of uncertainties is therefore correct for a regression of 'heart rate on step-up rate'.

Q4.18

If the test is investigating **whether there is, or is not, a relationship** between the two variables, you should use a test for **correlation**.

If you are measuring the **strength of a known relationship** (i.e. the slope of the best-fit line), then you should use a **regression analysis**

i) Correlation
You are investigating whether a relationship exists.

ii) Regression
You already know that oxygen increases with body weight, and you are carrying out the experiment to find out by how much the oxygen increases.
You are trying to measure the values of 'slope' and 'constant' in an equation of the form:

Oxygen consumption = Slope $\times$ (Body Weight) + Constant

iii) Correlation
You are investigating whether a relationship exists.

iv) Correlation
You are investigating whether a relationship exists.

Q4.19

i) The equation is: $P = 1.235Q + 2.845$

ii) The graph of the original data, plus the Line of Regression:

iii) $P = 1.235Q + 2.845$ and for $Q = 3.4$,

$P = 1.235 \times 3.4 + 2.845 = 7.044$

Q4.20

Excel Using the functions SLOPE and INTERCEPT:

SLOPE = 0.0308 INTERCEPT = 0.051 so the equation is

$A = 0.0308C + 0.051$

Minitab Transpose the data into columns:

Using Stat > Regression >Regression, with C as column 1 (C1) and A as column 2 (C2) results in

C2 = 0.0510 + 0.0308 C1

i) Slope = 0.0308

ii) Intercept = 0.0510

Q4.21

In each case the same result for the slope is obtained, with maybe a different number of significant figures.

i) to iv) Slope = 0.033444

Q4.22

As pure water gives no rotation, we would expect an equation of the form

$\theta = mw$ with the intercept $c = 0$

The data can be put into EXCEL and one of the methods used where the intercept can be forced to 0.

The LINEST function gives the slope 0.87878 to 5 decimal places

i) $\theta = 0.8788\ w$

ii) $4.9 = 0.8788\ w$
$w = 4.9 / 0.8788 = 5.58$ g

Q4.23

i) No – see Fig 4.7(a). The line is not straight

ii) Yes - see Fig 4.7(b). The line is straight

iii) The equation for Fig 4.7(b) is:

$$p = RT \times \frac{1}{V}$$

which is of the form
$y = mx + c$
with p plotted as y and $1/V$ plotted as x and $c = 0$.
The slope is then given by:
$m = RT$

iv) Intercept, $c = 0$ - see answer to (iii)

v) The theoretical equation has a zero intercept ($c = 0$), so the regression line should be forced through the origin.

vi) The slope, m, is calculated from the graph, and, provided that T is known, R can be calculated from:
$R = m/T = 2500/300 = 8.33\ J\ K^{-1}\ mol^{-1}$

vii) If the temperature, T, were to increase the slope, m (= RT), of the graph would also increase.
The intercept would stay zero.

Q4.24

i) Plot $(1/v)$ on the y-axis and $(1/S)$ on the x-axis

ii) With $(1/v)$ on the y-axis and $(1/S)$ on the x-axis, the coefficients of the straight line become:
Slope, $m = K_M / v_{max}$
Intercept, $c = 1/ v_{max}$

Hence $v_{max} = 1/c$ and then
$K_M = m \times v_{max} = m / c$