Essential Mathematics & Statistics for Science: Second Edition by Graham Currell & Antony Dowman (John Wiley & Sons)

Answers to 'Q' Questions

3 Equations in Science

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Q3.1

In each case put $x = 2$ and $y = 3$

i) $3x^2 = 3 \times 2^2 = 3 \times 4 = 12$

ii) $(3x)^2 = (3 \times 2)^2 = 6^2 = 36$

iii) $4(y - x) = 4(3 - 2) = 4(1) = 4 \times 1 = 4$

iv) $\frac{x}{y^2} = \frac{2}{3^2} = \frac{2}{9} = 0.222..$

v) $4(y - x^2) = 4(3 - 2^2) = 4(3 - 4) = 4(-1) = 4 \times (-1) = -4$

vi) $2xy^2 = 2 \times 2 \times 3^2 = 2 \times 2 \times 9 = 36$

vii) $xy - yx = 2 \times 3 - 3 \times 2 = 6 - 6 = 0$ ($xy = yx$ as order of multiplication does not matter)

viii) $\frac{x+y}{x} = \frac{x+y}{x} = \frac{2+3}{2} = \frac{5}{2} = 2.5$

Q3.2

$u = 5\ \text{ms}^{-1}$, $a = 3\ \text{ms}^{-2}$, $t = 4$ s
using [3.1} $v = u + at$
$v = 5 + 3 \times 4 = 5 + 12 = 17\ \text{ms}^{-1}$

Q3.3

$u = 30\ \text{ms}^{-1}$, $a = -5\ \text{ms}^{-2}$, $t = 6$ s
using [3.1} $v = u + at$
$v = 30 + (-5) \times 6 = 30 - 30 = 0\ \text{ms}^{-1}$

Q3.4

If $t = 5$, the equation could be written as $v = u + a5$	False, the 5 would come before the a, $v = u + 5a$

If $t = 5$, the equation could be written as $v = u + a \times 5$	True but would normally be written as $v = u + 5a$
If $t = 5$, the equation could be written as $v = u + 5a$	True
If $a = 1$, the equation could be written as $v = u + t$	True, the 1 is not needed.

Q3.5

i) $3(2 + x) = 6 + 3x$

ii) $3(2 + 4x) = 6 + 12x$

iii) $- 2(4x - 3) = -8x + 6 = 6 - 8x$

iv) $p(x + 2) = px + 2p$

v) $-3p(2 - x) = -6p + 3px = 3px - 6p$

vi) $p(x + p) = px + p^2$

Q3.6

i) $(v - 2)(v - 2) = v(v - 2) - 2(v - 2) = v^2 -2v \; -2v -2(-2)$
$= v^2 -2v \; -2v + 4 = v^2 -4v + 4$

ii) $(v - t)(v + t) = v(v + t) - t(v + t) = v^2 +vt \; -tv \; - t (+t)$
$= v^2 +vt \; -tv \; - t^2 = v^2 - t^2$

iii) $(v + 4)(3 - vt) = v(3 - vt) + 4(3 - vt) = 3v - v^2t +12 - 4vt$

iv) $(v + t)(3x + y) = v (3x + y) + t(3x + y) = 3xv + vy + 3tx + ty$

v) $(x - 2y)(p - q) = x(p - q) -2y(p - q) = px - qx \; - 2py + 2qy$

vi) $(v - t + 2)(3 + v + t) = v(3 + v + t)- t(3 + v + t) + 2(3 + v + t)$
$= 3v + v^2 + vt -3t \; - vt - t^2 + 6 + 2v + 2t = 5v + v^2 - t^2 + 6 - t$

Q3.7

See EXCEL file

i) $N = n + 3$
If $n = 6$, $N = 6 + 3 = 9$

ii) $N = 2n + 3$
If $n = 2$, $N = 2 \times 2 + 3 = 4 + 3 = 7$

iii) $N = (n + 3) \times 2 = 2(n + 3)$
If $n = 2$, $N = 2(2 + 3) = 2 \times 5 = 10$

iv) $N = \{2(n + 3) - 6\}/2$
If $n = 2, 6, 11$ $\quad$ $N = 2, 6, 11$
Note that the answer, N, is always the number, *n*, you first thought of!
We can prove that this is true by simplifying the equation:
$N = \{2(n + 3) - 6\}/2 = \{2n + 6 - 6\}/2 = 2n/2 = n$.

Q3.8

i) $A = (1/2)bh$

ii) $d = vt$

iii) $BMI = \frac{m}{h^2}$

iv) $V = (4/3)\,\pi r^3 = \frac{4\pi r^3}{3}$

v) $v = \sqrt{2gh}$

Q3.9

i) $4x + 4y = 4(x + y)$ so factors are 4 and $(x + y)$

ii) $4x + 6y = 2(2x + 3y)$ so factors are 2 and $(2x + 3y)$

iii) $4x + 6x^2 = 2x(2+3x)$ so factors are 2, x, and $(2 + 3x)$

iv) $pqx + pbx = px(q + b)$ so factors are p, x, and $(q+b)$

Q3.10

i) $x = \frac{4a}{2b} = \frac{\cancel{4}^2 a}{\cancel{2}^1 b} = \frac{2a}{b}$

ii) $x = \frac{4ap}{2pb} = \frac{\cancel{4}^2 ap}{\cancel{2}^1 pb} = \frac{2a\cancel{p}}{\cancel{p}b} = \frac{2a}{b}$

iii) It is not possible to cancel the '2' with the '4' because '4' is not a factor of the numerator, but y is a factor of the numerator and denominator:

$$x = \frac{y(4a+1)}{2by} = \frac{\cancel{y}(4a+1)}{2b\cancel{y}} = \frac{(4a+1)}{2b}$$

iv) Both 2 and y are common factors of the numerator and denominator:

$$x = \frac{2y(2a+3)}{4by} = \frac{\cancel{2}^1 \cancel{y}(2a+3)}{\cancel{4}^2 b\cancel{y}} = \frac{(2a+3)}{2b}$$

Q3.11

i) The a's are factors of both top and bottom and can be cancelled: Yes

ii) You can cancel the '2' on top with the '2' underneath: Yes

iii) The '+2's on top and bottom are not factors and can not be cancelled: No

iv) Dividing by 'a' on top is the same as multiplying by 'a' on the bottom: Yes

v) Yes

vi) Yes

Q3.12

i) $8k \mathbf{+ 2x} = 16 - 2x \mathbf{+ 2x} = 16$

ii) $8k \mathbf{+} 2x \mathbf{- 8k} = 16 \mathbf{- 8k}$

$2x \mathbf{=} 16 - 8k$

iii) $\frac{2x}{2} = \frac{16-8k}{2}$ gives $x = 8 - 4k$

iv) Yes

Q3.13

i) $\frac{x}{4} + 2y - 2y = 3 - 2y$ so $\frac{x}{4} = 3 - 2y$

ii) $\frac{x}{4} \times 4 = (3-2y) \times 4 = 4(3-2y)$

Cancelling the 4's on the LHS

$x = 4(3\text{-}2y) = 12 - 8y$

iii) Yes

Q3.14

i) $x = -4 + 2p$ or $x = 2p - 4$

ii) $x = -3(2 - \mu) + 4p$

iii) $x = -8 + (q - t)$

Q3.15

i) $18 = p - x$ *add x to both sides*

$x + 18 = p - x + x = p$ *subtract 18 from both sides*

$x = p - 18$

ii) $3(q - p) = 10 - x$ *add x to both sides*

$x + 3(q - p) = 10$ *subtract* $3(q - p)$ *from both sides*

$x = 10 - 3(q - p)$

iii) $9 = x - p$ *swop sides*

$x - 9 = p$ *add 9 to both sides*

$x = p + 9$

iv) $(2v - t) = 8 + x - k$ *swop both sides*

$8 + x - k = (2v - t)$ add k and subtract 8 from both sides

$x = (2v - t) - 8 + k$

Q3.16

i) Dividing by 2

$x = 30 + 10p$

ii) Multiply by 4

$x = 4(5 - 2q) = 20 - 8q$

iii) Divide by (s-m)

$$x = \frac{k}{s - m}$$

iv) Multiply by (2p-q)

$x = 8(2p - q) = 16p - 8q$

Q3.17

i) x = 5 – 4 = 1

ii) x = 4 – 6 = -2

iii) x = 6 – a = 6 – 2 = 4

iv) $3 - a = 4 - x$

$x + 3 - a = 4$

$x = 4 - 3 + a$ = 1 + a = 1 + 2 = 3

v) $a + x - 3 = 4$

x – 3 = 4 –a

x = 4 –a + 3 = 7 – a = 7 - 2 = 5

vi) $25 - b - c = 30 - x$ change all signs

$-25 + b + c = -30 + x$ swop sides

$-30 + x = -25 + b + c$ move 30

x = $-25 + b + c + 30 = 5 + 8 + 1$ = 14

Q3.18

i) Subtract p from both sides

$-x = 6 - p$

Multiply by -1

x* = -6 + p** or ***x* = p - *6

ii) Add 4 to both sides

$3 - k + 4 = x$

$7 - k = x$

Swap both sides

***x* = 7- k**

iii) $3 - x = 0$

$3 = x$ so $x = 3$

iv) $3p - 4 = 4 + x + 2p$ Subtract 2p from both sides

$p - 4 = 4 + x$ Subtract 4 from both sides

$p - 8 = x$ so $x = p - 8$

Q3.19

i) $5x = k - 6$

Divide both sides by 5

$x = \frac{k-6}{5}$ — note that the whole of (*k*-6) is divided by 5

ii) $mx = y - c$ — Divide both sides by m

$x = \frac{y-c}{m}$

iii) $\frac{x}{3} = p - 4$ — Multiply both sides by 3

$x = 3(p - 4) = 3p - 12$

iv) $(2 - p)\,x = p + 4$ — Divide both sides by (2-*p*)

$x = \frac{p+4}{2-p}$

v) $\frac{x}{3-p} = 5$ — Multiply both sides by (3 - *p*)

$x = 5(3 - p)$

vi) $\frac{(2-p)x}{3-p} = 2$ — Do this in 2 stages.

Multiply both sides by (3 – *p*)

$(2 - p)x = 2(3 - p)$ — Now divide both sides by (2 – *p*)

$x = \frac{2(3-p)}{2-p}$ — do not cancel 2 or *p*!

vii) $8 = \frac{p+q}{x}$ — multiply both sides by *x*

$8x = p + q$ — divide by 8

$x = \frac{p+q}{8}$

viii) $(p-4) = \frac{5}{x}$ — multiply both sides by *x*

$(p - 4)\,x = 5$ — divide by (*p* – 4)

$x = \frac{5}{p-4}$

Q3.20

$pV = nRT$ — *R* is a fundamental constant

In Charles's Law both *p* and *n* are constants as it is a fixed amount of gas at constant pressure, so the only variables are *V* and *T*.

Q3.21

i) $pH = -\log([H^+])$

log is logarithm to the base 10.

$[H^+]$ is a chemical notation for the concentration of hydrogen ions. (See 5.1.9)

ii) $A_t = A_O \exp(-0.693\, t / T_{½})$

$\exp(-0.693\, t / T_{½})$ is the same as $e^{(-0.693\, t / T½)}$

The subscripts for A represent the values of A at those times, e.g. A_t is the value of A at time t, A_0 is the value of A at time 0.

The subscript '½' in $T_{½}$ refers to the half life time.

iii) $$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

χ is the Greek letter 'chi' and is used here for the 'chi squared' value in statistics.

Σ is the Greek letter sigma and refers to a summation over i terms. For example, if i = 3, then

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^2}{E_3}$$

Q3.22

i) $x^2 = 25$ Inverse of 'square' is 'square root'
$x = \sqrt{25} = \pm 5$ (the '±' symbol indicates that the square root of 25 could be either '+5' or '-5')

ii) $x^2 = 0.025$ Inverse of 'square' is 'square root':
$x = \sqrt{0.025} = \pm 0.158$

iii) $\sqrt{x} = 9$ Inverse of 'square root' is 'square':
$x = 9^2 = 81$

iv) $\log(x) = 1.3$ Inverse of 'log' is 'power of 10':
$x = 10^{1.3} = 19.95$

v) $e^x = 26.2$ Inverse of 'e' is 'ln':
$x = \ln(26.2) = 3.27$

vi) $\sin(x) = 0.34$ Inverse of 'sin' is '$\sin^{-1}$':
$x = \sin^{-1}(0.34) = 19.88$ degrees

vii) $10^x = 569$ Inverse of 'power of 10' is 'log':
$x = \log(569) = 2.755$

viii) $10^x = 0.01$ Inverse of 'power of 10' is 'log':
$x = \log(0.01) = -2$

Q3.23

i) $2x - 8 = 22$ add 8 to both sides
$2x = 22 + 8 = 30$ divide by 2
$x = 15$

ii) $3(2 - x) = 12$ divide by 3
$2 - x = 4$ subtract 2 from both sides
$-x = 2$ change signs (multiply by -1)
$x = -2$

iii) $k = 5 - px$ multiply by -1
$-k = -5 + px$ move 5 across
$-k + 5 = px$ divide by p

$$\frac{-k+5}{p} = x \quad \text{or} \quad x = \frac{5-k}{p}$$

iv) $q(2-x) = v$ multiply out brackets

$2q - qx = v$ multiply by -1

$-2q + qx = -v$ move $-2q$

$qx = 2q - v$ divide by q

$x = \frac{2q - v}{q}$ or $x = 2 - \frac{v}{q}$

Q3.24

i) $x^2 + t = 25$ move t

$x^2 = 25 - t$ take square root of both sides

$x = \sqrt{(25 - t)}$

ii) $(x + t)^2 = 25$ square root of both sides

$x + t = \sqrt{25}$ move t over

$x = \sqrt{25} - t$ note that x could be $(5 - t)$ or $(-5 - t)$

iii) $0.6k = \sin(2x)$ change sides

$\sin(2x) = 0.6k$ take inverse sin

$2x = \sin^{-1}(0.6k)$ divide by 2

$x = \frac{\sin^{-1}(0.6k)}{2}$

iv) $y = a - \frac{4}{x}$ multiply by x

$xy = ax - 4$ gather x terms together and move 4

$4 = ax - yx$ factorise

$4 = (a-y)x$ divide by (a-y)

$\frac{4}{a-y} = x$

v) $y = e^{2x}$ take 'ln' of both sides

$\ln(y) = 2x$ divide by 2 and swop sides

$x = \frac{\ln(y)}{2}$

vi) $y = \sqrt{2x}$ square both sides

$y^2 = 2x$ divide by 2

$\frac{y^2}{2} = x$

vii) $y = 3 \times \sqrt{(2x - 7)}$ divide by 3

$\frac{y}{3} = \sqrt{(2x - 7)}$ square both sides

$\frac{y^2}{9} = 2x - 7$ move 7

$\frac{y^2}{9} + 7 = 2x$ divide by 2

$\frac{\frac{y^2}{9} + 7}{2} = x$ or $x = \frac{y^2}{18} + \frac{7}{2}$

viii) $y = \sqrt{\left(\frac{2}{x} + k\right)}$ square both sides

$y^2 = \frac{2}{x} + k$ move k

$y^2 - k = \frac{2}{x}$ multiply by x

$x(y^2 - k) = 2$ divide by y^2 - k

$x = \frac{2}{y^2 - k}$

Q3.25

i) $\log(x + 200) = 2.6$ inverse log

$x + 200 = 10^{2.6} = 398.1$ move 200

$x = 398.1 - 200 = 198.1$

ii) $\log(2x) + 0.3 = 2.5$ move 0.3

$\log(2x) = 2.5 - 0.3 = 2.2$ inverse log

$2x = 10^{2.2} = 158.49$ divide by 2

$x = \frac{158.5}{2} = 79.24$

iii) $e^x = 26$ take 'ln' of both sides

$x = \ln(26) = 3.258$

iv) $e^{3x} = 0.87$ take 'ln' of both sides

$3x = \ln(0.87)$ divide by 3

$x = \frac{\ln(0.87)}{3} = \frac{-0.139}{3}$ = -0.0464

v) $\log(2x + 1.2) = 0.45$ inverse 'log'

$(2x + 1.2) = 10^{0.45} = 2.818$ move 1.2

$2x = 2.818 - 1.2 = 1.618$ divide by 2

$x = \frac{1.618}{2}$ = 0.809

vi) $10^{x+2} = 246$ take logs

$x + 2 = \log(246) = 2.39$ move 2 across

$x = 2.39 - 2 = 0.39$

Q3.26

i) $F = \frac{q_1 q_2}{4\pi\varepsilon_O x^2}$ multiply by $4\pi\varepsilon_0 x^2$

$4\pi\varepsilon_0 x^2 F = q_1 q_2$ divide by $4\pi\varepsilon_0 F$

$x^2 = \frac{q_1 q_2}{4\pi\varepsilon_0 F}$ take square root

$x = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 F}}$

ii) $n = \frac{\sin(\theta_i)}{\sin(\theta_r)}$ multiply by $\sin(\theta_r)$

$n\sin(\theta_r) = \sin(\theta_i)$ divide by n

$\sin(\theta_r) = \frac{\sin(\theta_i)}{n}$ inverse sin

$$\theta_r = \sin^{-1}\left(\frac{\sin(\theta_i)}{n}\right)$$

iii) $E = E_O - \frac{RT}{zF}\ln\left(\frac{a_{red}}{a_{ox}}\right)$ move terms as shown….

$\frac{RT}{zF}\ln\left(\frac{a_{red}}{a_{ox}}\right) = E_0 - E$ multiply by *zF/RT*

$\ln\left(\frac{a_{red}}{a_{ox}}\right) = \frac{(E_0 - E)zF}{RT}$ inverse 'ln'

$\frac{a_{red}}{a_{ox}} = e^{\frac{(E_0 - E)zF}{RT}}$ multiply by a_{ox}

$$a_{red} = a_{ox}e^{\frac{(E_0 - E)zF}{RT}}$$

iv) $pV^{\gamma} = K$ divide by *p*

$V^{\gamma} = \frac{K}{p}$ take 'log' of both sides

$\gamma\log(V) = \log\left(\frac{K}{p}\right)$ divide by log*(V)*

$$\gamma = \frac{\log\left(\frac{K}{p}\right)}{\log(V)}$$

Q3.27

i) $2x + 4 = x - 2$ subtract x from both sides

$2x + 4 - x = x - 2 - x$

$x + 4 = -2$ and then subtract 4 from both sides

$x + 4 - 4 = -2 - 4$

$x = -6$

ii) $3 - a + x = 6 - 2x$ add $2x$ to both sides

$3 - a + x + 2x = 6 - 2x + 2x$

$3 - a + 3x = 6$ and now subtract $(3 - a)$ from both sides

$3 - a + 3x - (3 - a) = 6 - (3 - a)$

$3 - a + 3x - 3 + a = 6 - 3 + a$

$3x = 3 + a$ and finally divide by 3

$$x = \frac{3 + a}{3}$$

iii) $x - 3 - 2p = 5 + p - 3x$

In a similar way to ii) above, add 3x and then add 3+ 2p to both sides

$x - 3 - 2p + 3x + 3 + 2p = 5 + p - 3x + 3x + 3 + 2p$

$4x = 8 + 3p$

$x = \frac{8+3p}{4} = 2 + \frac{3p}{4}$

iv) $x = 2(p - x)$ multiply out the bracket

$x = 2p - 2x$ and then add 2x to both sides

$x + 2x = 2p - 2x + 2x$

$3x = 2p$

$x = \frac{2p}{3}$

v) $x(2 - a) = a(3 - x)$ multiply out both brackets

$2x - ax = 3a - ax$ and add ax to both sides (or simply cancel ax from both sides)

$2x - ax + ax = 3a - ax + ax$

$2x = 3a$

$x = \frac{3a}{2}$

vi) $2(x - 1) = 3(3 - x)$ multiply out both brackets

$2x - 2 = 9 - 3x$ and add 3x to both sides

$2x - 2 + 3x = 9 - 3x + 3x$

$5x - 2 = 9$ and finally add to both sides and then divide by 5

$5x - 2 + 2 = 9 + 2$

$5x = 11$

$x = \frac{11}{5}$

Q3.28

i) $5x - 2x = 3 \times x = 3x$

ii) $3px + 2px = 5p \times x = 5px$

iii) $mx - 3x = (m - 3) \times x = (m - 3)x$

iv) $5x - px - 2x = (5 - p - 2) \times x = (3 - p)x$

v) $5x - px - 7x = (5 - p -7) \times x = (-2 - p) \times x = -(2 + p)x$

Note that in the last step we can put a 'minus' outside the whole bracket, provided that we change the sign of every term inside the bracket.

vi) $3x + px - 4x + 2kx = (3 + p - 4 + 2\mathrm{k}) \times x = (p -1 + 2k)x$

Q3.29

i) $3(2 + x) - 2x + 4 = 6 + 3x - 2x + 4 = \mathbf{10 + \boldsymbol{x}}$

ii) $3(2 + x) + 2(3x + 1) = 6 + 3x + 6x + 2 = \mathbf{8 + 9\boldsymbol{x}}$

iii) $2(3 - 3x) - 4(1 - 4x) = 6 - 6x - 4 + 16x = \mathbf{2 + 10\boldsymbol{x}}$

iv) $a(b - 2x) + b(2x - a) = ab - 2ax + 2bx - ba = - 2ax + 2bx$

$= -2x(a - b) = -2(a - b)x = \mathbf{2(b - a)x}$

Note:

In the second step: $ab - ba = ab - ab = 0$

In the last step it was convenient to change the signs of both '*a*' and '*b*' *inside* the bracket to remove the negative sign *outside* the bracket.

v) $x(2 + a) - 3(a - x) = 2x + ax - 3a + 3x = 5x + ax - 3a = \mathbf{(5 + a)x - 3a}$

vi) $2(a - 4) - 4(a - 2 - x) + 2a - 3x = 2a - 8 - 4a + 8 + 4x + 2a - 3x = \mathbf{x}$

Q3.30

i)

$2 + x = 5 – x$

$2 + x + x = 5 - x + x$

$2 + 2x = 5$

$2x = 5 - 2 = 3$

$x = 3/2 = 1.5$

ii)

$2x –a = b + 4x$

$2x – 4x = b + a$

$-2x = b + a$

$x = (b + a)/(-2)$ or $-(a + b)/2$

iii)

$x = 3(2 - x)$

$x = 6 – 3x$

$4x = 6$

$x = 6/4 = 3/2 = 1.5$

iv)

$3x + 3 = px + 8$

$3x - px = 8 - 3$

$(3 - p) x = 5$

$x = 5 / (3 - p)$

v)

$3x + 3 = px + 8 - p$

$3x - px = 8 - 3 - p$

$(3 - p) x = 5 - p$

$x = (5 - p) / (3 - p)$

vi)

$4 - qx + p = 2x - px + 9$

$-qx + px -2x = 9 - 4 - p$

$(p - q -2) x = 5 - p$

$x = (5 - p) / (p - q -2)$

vii)

$x(1 – 2a) = 4a(x – 3)$

$x – 2ax = 4ax – 12a$

$x – 6ax = -12a$

$x(1 – 6a) = -12a$

$x = -12a/(1 - 6a)$ or $12a/(6a - 1)$

viii)

$(2 – x)p = 6$

$2p – px = 6$

$2p – 6 = px$

$x = (2p -6)/p$

ix) $(2 - x)p = 6 - x$

$2p - px = 6 - x$

$2p - 6 = px - x = x(p - 1)$

$x = (2p - 6)/(p - 1)$

Q3.31

i) $2 = \frac{4}{x}$ multiply both sides by *x* and cancel *x*'s on the RHS

$2 \times x = \frac{4}{x} \times x$ cancel *x*

$2x = 4$ divide each side by 2

$x = 2$

ii) $2 = \frac{4 - a}{x}$ multiply both sides by *x* and cancel *x*'s on the RHS

$2 \times x = \frac{(4 - a)}{x} \times x$ cancel *x*

$2x = 4 - a$ divide each side by 2

$x = \frac{4 - a}{2}$

iii) $2 = \frac{4}{x} - a$ multiply all terms by *x* and cancel *x*'s in one place

$2 \times x = \frac{4}{x} \times x - ax$

$2x = 4 - ax$ collect the *x*'s on LHS

$2x + ax = 4$ find the number of *x*'s

$x(2 + a) = 4$ divide by 2+*a*

$x = \frac{4}{2 + a}$

iv) $\frac{1}{x} + \frac{1}{a} = 3$ Multiply both sides by x, and cancel the x's in the first term:

$$\frac{\not{x}^1}{\not{x}^1} + \frac{x}{a} = 3x$$

Multiply both sides by *a*:

$$1 + \frac{x}{a} = 3x$$

$$a + \frac{x \times \not{a}}{\not{a}} = 3xa$$

Collect the x terms on the LHS:

$$a + x = 3xa$$

x - 3xa = -a which gives

(1 - 3a)x = -a Dividing both sides by (1 - 3a):

$$x = -\frac{a}{(1-3a)} = \frac{a}{(3a-1)}$$

NB The negative sign was removed in the last stage by multiplying the bracket term in the denominator by '-1'.

v) $$\frac{1}{x+1} + \frac{1}{a} = 3$$

Multiply both sides by x+1, and cancel the (x+1)'s in the first term:

$$\frac{(x+1)}{x+1} + \frac{(x+1)}{a} = 3(x+1)$$

$$1 + \frac{(x+1)}{a} = 3x + 3$$

$$\frac{(x+1)}{a} = 3x + 2$$

Multiply both sides by a:

$$\frac{(x+1) \times \not{a}}{\not{a}} = 3xa + 2a$$

$$x + 1 = 3xa + 2a$$

Collecting x terms on LHS

x(1 - 3a) = 2a – 1

Dividing both sides by (1-3a)

$$x = \frac{(2a-1)}{(1-3a)}$$

vi) $$\frac{x}{x+1} + \frac{1}{a} = 3$$

Multiply both sides by x+1, and cancel the (x+1)'s in the first term:

$$\frac{x(x+1)}{x+1} + \frac{(x+1)}{a} = 3(x+1)$$

$$x + \frac{(x+1)}{a} = 3x + 3$$

Multiply both sides by *a*:

$$\frac{(x+1)}{a} = 2x + 3$$

$$\frac{(x+1)\times \not{a}}{\not{a}} = 2xa + 3a$$

Collecting *x* terms on LHS

$$x + 1 = 2xa + 3a$$

$x(1 - 2a) = 3a – 1$ Dividing both sides by (1-2*a*)

$$x = \frac{(3a - 1)}{(1 - 2a)}$$

vii) $\frac{3}{x+1} = \frac{4a}{x-2}$ Multiply both sides by (*x*+1) and (*x*-2):

$\frac{3(x+1)(x-2)}{x+1} = \frac{4a(x+1)(x-2)}{x-2}$ Cancelling terms gives

$3(x - 2) = 4a(x + 1)$ Multiplying out the brackets

$3x - 6 = 4ax + 4a$ Collecting *x* terms

$3x - 4ax = 4a + 6$

$x(3 - 4a) = 4a + 6$ Dividing both sides by (3 - 4*a*)

$$x = \frac{4a + 6}{(3 - 4a)}$$

viii) $\frac{x+1}{x-1} = y$ Multiply both sides by *x* - 1

$x + 1 = y(x - 1)$

$x + 1 = yx – y$ Collecting *x* terms

$x - yx = -y - 1$

$x(1 - y) = -(y + 1)$ Dividing both sides by (1-*y*)

$$x = \frac{-(y+1)}{(1 - y)} = \frac{y+1}{y-1}$$

Q3.32

In these questions it is necessary rearrange the equation into the form:
$ax^2 + bx + c = 0$
and then, by comparison with this standard equation, derive the values for the constants *a*, *b* and *c*.
These values can then be substituted into the quadratic equation formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i) $2x^2 - 3x + 1 = 0$

This equation is already in the standard form, and we see directly that:

$a = 2$, $b = -3$ and $c = 1$

Substituting into the formula:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} = \frac{4}{4} \quad oor \frac{2}{4}$$

The solutions to the equation are therefore:

$x = 1$ or 0.5

ii) $3.2x^2 - 2.5x - 0.8 = 0$

This equation is already in the standard form, and we see directly that:

$a = 3.2$, $b = -2.5$ and $c = -0.8$

Substituting into the formula:

$$x = \frac{-(-2.5) \pm \sqrt{(-2.5)^2 - 4 \times 3.2 \times (-0.8)}}{2 \times 3.2} = \frac{2.5 \pm \sqrt{6.25 + 10.24}}{6.4}$$

$$= \frac{2.5 \pm \sqrt{16.46}}{6.4} = \frac{2.5 \pm 4.057}{6.4} = \frac{6.557}{6.4} or \frac{-1.557}{6.4}$$

This gives two possible solutions:

$x = 1.025$ or -0.244

iii) $4x = 5 - 2x^2$

Rearrange the equation into the standard form:

$2x^2 + 4x - 5 = 0$

Comparing with the standard equation, we can see that:

$a = 2$, $b = 4$ and $c = -5$

Substituting into the formula:

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 2 \times (-5)}}{2 \times 2} = \frac{-4 \pm \sqrt{16 + 40}}{4} = \frac{-4 \pm 7.4833}{4}$$

This gives two possible solutions:

$x = -2.87$ or 0.871

iv) $(x - 2)^2 = 2x$

Multiply out the bracket:

$x^2 - 4x + 4 = 2x$

which can be rearranged to give

$x^2 - 6x + 4 = 0$

Comparing with the standard equation, we can see that:

$a = 1$, $b = -6$ and $c = 4$

Substituting into the formula:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times 4}}{2 \times 1} = \frac{6 \pm \sqrt{36 - 16}}{2} = \frac{6 \pm 4.47}{2}$$

This gives two possible solutions:

$x = 5.236$ or 0.764

Q3.33

i) $x^2 - 4x + 4 = 0$

This equation is already in the standard form, and we see directly that:
$a = 1$, $b = -4$ and $c = 4$
Substituting into the formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 4}}{2 \times 1} = \frac{4 \pm \sqrt{16 - 16}}{2} = \frac{4 \pm 0}{2} = 2 \qquad \square$$

Since $b^2 = 4ac$, the two solutions to the equation are identical, giving only one actual solution:

$x = 2$

ii) $x^2 + 0.5x – 1.5 = 0$

This equation is already in the standard form, and we see directly that:
$a = 1$, $b = 0.5$ and $c = -1.5$
Substituting into the formula:

$$x = \frac{-(0.5) \pm \sqrt{(0.5)^2 - 4 \times 1 \times -(1.5)}}{2 \times 1} = \frac{-0.5 \pm \sqrt{0.25 + 6}}{2} = \frac{-0.5 \pm 2.5}{2} = \frac{-3}{2} \text{ or } \frac{2}{2}$$

$x = -1.5$ or $x = 1$

iii) $2x^2 - 3x + 4 = 0$

Comparing with the standard equation, we can see that:
$a = 2$, $b = -3$ and $c = 4$
Substituting into the formula:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 4}}{2 \times 2} = \frac{3 \pm \sqrt{9 - 32}}{4} = \frac{3 \pm \sqrt{-23}}{4} \qquad ????$$

It is not possible to get a real solution to the square root of a negative number: $b^2 < 4ac$.
There are no real solutions to this equation. There is no real value of x that will make the equation true.

iv) $4x^2 + 12x + 9 = 0$

This equation is already in the standard form, and we see directly that:
$a = 4$, $b = 12$ and $c = 9$
Substituting into the formula:

$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4 \times 4 \times 9}}{2 \times 4} = \frac{-12 \pm \sqrt{144 - 144}}{8} = \frac{-12 \pm 0}{8} = \frac{-3}{2}$$

Since $b^2 = 4ac$, the two solutions to the equation are identical, giving only one actual solution:

$$x = \frac{-3}{2}$$

Q3.34

i) $d = 500 + 1.2t$
$d = 3200 - 0.9t$

Hence we can combine the two equations to give:

$500 + 1.2t = 3200 - 0.9t$
Rearrange to get t terms on the LHS:
$1.2t + 0.9t = 3200 - 500$
$2.1t = 2700$

giving

$t = 2700 / 2.1 = 1285.7$ s

ii) Substituting the above value of t into the first equation:
$d = 500 + 1.2 \times 1.285.7 = 2042.9$ m

Q3.35

The first step is to rearrange the equations so that one of the variables is the subject of both equations.
We chose to rearrange the equations to collect C_1 terms on the LHS:

$2.25 \times C_1 = 0.773 - 2.0 \times C_2$
and
$0.25 \times C_1 = 0.953 - 6.0 \times C_2$

and then dividing the two equations by 2.25 and 0.25 respectively:

$C_1 = (0.773/2.25) - (2.0/2.25) \times C_2 = 0.3436 - 0.8889 \times C_2$ (A)
and
$C_1 = (0.953/0.25) - (6.0/0.25) \times C_2 = 3.812 - 24.0 \times C_2$ (B)

So $0.3436 - 0.8889C_2 = 3.812 - 24.0\ C_2$

$24.0 \times C_2 - 0.8889 \times C_2 = 3.812 - 0.3436$

$23.111\ C_2 = 3.4684$

$C_2 = 0.1500$

Substituting in equation A

$C_1 = 0.3436 - 0.8889 \times 0.1500 = 0.210$

Q3.36

If $y = x^2$ and $y = 1.5 - 0.5x$

Then $x^2 = 1.5 – 0.5x$ and this can be rearranged to give

$x^2 + 0.5x – 1.5 = 0$

$a = 1$, $b = 0.5$, $c = -1.5$ and substituting in the formula..

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0.5 \pm \sqrt{(-0.5)^2 - 4 \times 1 \times (-1.5)}}{2}$$

$$x = \frac{-0.5 \pm \sqrt{0.25 + 6}}{2} \quad \text{and} \quad x = \frac{-0.5 \pm \sqrt{6.25}}{2} \;, \quad x = \frac{-0.5 \pm 2.5}{2}$$

$$x = \frac{2}{2} = 1 \quad \text{or} \quad x = \frac{-3}{2} == -1.5$$

Substituting back for x in say $y = x^2$ gives $y = 1$ or $y = 2.25$ which corresponds with the graphical solution of E3.46

Q3.37

The first step is to rearrange the equations so that one of the variables is the subject of both equations.

We chose to rearrange the equations to collect C_1 terms on the LHS:

$2.25 \times C_1 = 0.773 - 2.0 \times C_2$

and

$0.25 \times C_1 = 0.953 - 6.0 \times C_2$

and then dividing the two equations by 2.25 and 0.25 respectively:

$C_1 = (0.773/2.25) - (2.0/2.25) \times C_2 = 0.3436 - 0.8889 \times C_2$

and

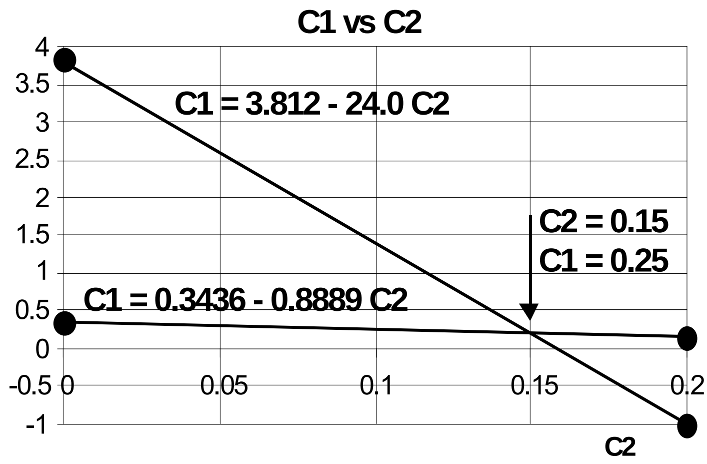
$C_1 = (0.953/0.25) - (6.0/0.25) \times C_2 = 3.812 - 24.0 \times C_2$

values for C_1 are calculated for values of $C_2 = 0$ and 0.2, giving

$C_1 = 0.3436$ and $C_1 = 0.16582$, and

$C_1 = 3.812$ and $C_1 = -0.988$.

Straight lines drawn between these points (as in Fig below) and the lines appear to cross when (approximately) time, $C_1 = 0.25$ and distance, $C_2 = 0.15$.



By recalculating values for C_1 for values of C_2 = 0145 and 0.155, giving C_1 = 0.2147 and C_1 = 0.2058, and
C_1 = 0.332 and C_1 = 0.092.

Straight lines drawn between these points (as in Fig below) and the lines appear to cross when (approximately) time, C_1 = 0.21 and distance, C_2 = 0.150.