Essential Mathematics & Statistics for Science: Second Edition by Graham Currell & Antony Dowman (John Wiley & Sons)

Answers to 'Q' Questions

2 Scientific Data

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Q2.1

i) $42600 = 4.26 \times 10^4$

ii) $0.00362 = 3.62 \times 10^{-3}$

iii) $10000 = 1.0 \times 10^4$

iv) $0.0001 = 1.0 \times 10^{-4}$

v) $0.045 \times 10^4 = 4.5 \times 10^2$

vi) $26.6 \times 10^3 = 2.66 \times 10^4$

vii) $3.2E3 = 3.2 \times 10^3$

viii) $4.5E\text{-}6 = 4.5 \times 10^{-6}$

Q2.2

i) $120000 \times 0.003 = 360 = 3.6 \times 10^2$

ii) $5.0 \times 10^5 \times 3.0 \times 10^{-3} = 15 \times 10^{5-3} = 15 \times 10^2 = 1.5 \times 10^3$

iii) $\frac{1.2\times10^5}{3.0\times10^3} = 0.4\times10^{5-3} = 0.4\times10^2 = 4.0\times10^1$ probably left as 40

iv) $4500 \div 0.09 = 50000 = 5 \times 10^4$

v) $0.0056 \times 4.0 \times 10^3 = 2.24 \times 10^1$ probably left as 22.4

vi) $\frac{1.2\times10^5}{3.0\times10^{-3}} = 0.4\times10^{5-(-3)} = 0.4\times10^8 = 4.0\times10^7$

Q2.3

i) Make each term have the same 'power of ten':
$1.2463 \times 10^3 - 42.1 = 1.2463 \times 10^3 - 0.0421\times 10^3 = 1.2042 \times 10^3$

ii) Carry out the subtraction in the numerator

$$\frac{7.2463\times10^6 - 1.15\times10^5}{3.0\times10^{-3}} = \frac{72.463\times10^5 - 1.15\times10^5}{3.0\times10^{-3}} = \frac{71.313\times10^5}{3.0\times10^{-3}}$$

Then carry out the division:

$$\frac{71.313\times10^5}{3.0\times10^{-3}} = \frac{71.313}{3.0}\times\frac{10^5}{10^{-3}} = 23.771\times10^{5-(-3)} = 23.771\times10^8$$

and put the result into scientific notation:
$= 2.3771 \times 10^9$

Q2.4

i) 0.047

ii) 0.046

iii) 14.0 (the last '0' is significant and must be kept)

iv) 7.35×10^3

v) 27000 (the first '0' is significant, but the next two '0's are placeholders to show the correct power of ten)

vi) 11.3 (sometimes rounded to make that the last digit even, which would give 11.2)

vii) 11.2

viii) 5.65×10^{-3} (sometimes rounded to make that the last digit even, which would give 5.64×10^{-3})

Q2.5

i) 0.047

ii) 8.00 (both the last '0's are required to give the two decimal places)

iii) 426.89

iv) 1.35 (sometimes rounded to make that the last digit even, which would give 1.34)

Q2.6

The least accurate value is 3.10 g, which is only quoted to 2 decimal places - the final answer cannot claim any greater accuracy. Hence the final answer is also limited to 2 dp:
3.8891 g = 3.89 g

Q2.7

Approximations give:
Time = 5hr
Distance = 2500miles
Approximate speed = 2500 / 5 = 500 mph
The estimate for distance was a little low, giving a lower speed, and the estimate for time was a little high, also giving a lower apparent speed. We can conclude that our estimate is going to be on the low side - and an underestimate.
The exact speed is actually 523 (3sf) mph.

Q2.8

The three dots, '...', following a number indicates that your calculator will probably be showing more digits in the answer than are displayed here.

i) $1/(2.5 \times10^4) = 4.0 \times10^{-5}$

ii) $(-0.0025) \times (-1.2 \times10^{-6})$ = 2083.33...

iii) $3.2^{-1.6} = 0.1555$

iv) $3.487^2 = 12.159...$

v) Square root of 0.067 = 0.259 In fact the answer could be +0.259 *or* –0.259

Q2.9

$4.7\ km = 4.7 \times 10^3\ m$
$6.2\ nm = 6.2 \times 10^{-9}\ m$
The ratio between the two ~ $10^3 / 10^{-9} = 10^{12}$
This is 12 orders of magnitude.

Q2.10

i) From Conversion between Units:
1 kcal = 4.2 kJ
Food energy = 750 × 4.2 = 3150 kJ

ii) 4.2 kJ = 1 kcal
So 1 kJ = 1/(4.2) kcal = 0.2381 kcal
1200 kJ = 1200 × 0.2381 kcal = 285.7 kcal

iii) 1 kg = 2.20 lb
So 1lb = 1/2.2 kg
145 lb = 145/2.2 kg = 65.91 kg

iv) 1 inch = 25.4 mm
So 6 inches = 6 × 25.4 mm = 152.4 mm
A plank 6 inches by 1 inch is therefore 152.4 mm by 25.4 mm
(The standard size is 150 mm by 25 mm)

v) There are 2 conversions to do in this question.
1 gallon = 8 pints, 5 gallons = 5 × 8 pints = 40 pints
As 1 pint = 568 mL, 40 pints = 40 × 586mL = 22720 mL = 22.72 L
Then 5 gallons = 22.72 L

vi) 4 kcal = 1 g of protein
1 kcal = 1/4 g of protein
46 kcal = 46 ×1 / 4 g = 11.5 g of protein

Q2.11

i) $1\ km^2 = (1 \times 10^3\ m) \times (1 \times 10^3\ m) = 1 \times 10^6\ m^2$
1 hectare = $1 \times 10^4\ m^2$
so number of hectares in $1\ km^2 = (1 \times 10^6)/(1 \times 10^4) = 100$

ii) Calculate the answer in simple steps.
$1\ g = 1 \times 10^{-3}\ kg$
Hence
$7.9\ g\ cm^{-3} = 7.9 \times 10^{-3}\ kg\ cm^{-3}$

We now need to convert cm^3 to m^3
1 cm = 1×10^{-2} m
1 cm^3 = 1×10^{-6} m^3
We know that there are 7.9×10^{-3} kg in 1.0 cm^3
This is the same as saying that there are 7.9×10^{-3} kg in 1×10^{-6} m^3
The density is therefore:

$$\frac{7.9\times10^{-3}}{1.0\times10^{-6}} = 7.9\times10^{3}\ \mathrm{kg\ m^{-3}}$$

iii) 1 m^2 = 100 cm × 100 cm = 1×10^4 cm^2
every 1 cm^2 requires 0.015 g
so 1 m^2 = 1×10^4 cm^2 requires 0.015×10^4 g
Hence rate = $0.015 \times 10^4 / 1000$ kg m^{-2} = 0.15 kg m^{-2}

iv) 40 mpg in litres per 100 km
This is a change of 2 units of measurement and a different way of giving the fuel consumption.
40 miles = 40 × 1.61 km = 64.4 km
1 gallon = 4.55 L
So 4.55 L goes 64.4 km
4.55/64.4 L goes 1 km
100 × 4.55/64.4 L = 7.07 L goes 100 km
The fuel consumption is therefore 7.07 L per 100 km.

Q2.12

In 100 atoms, 80 will be mass 11 and 20 mass 10
Average mass = (80 ×11 + 20 ×10) /100 = 10.8

Q2.13

i) Since Relative Molecular Mass (*RMM*) of $C_9H_8O_4$ is 180
M_r = 180
ii) M_m = 180 g mol^{-1}
iii) Mass of 1 mole = 180 g

Q2.14

i) Since Relative Molecular Mass (*RMM*) of Na_2CO_3 is 106
Mass of 1 mole = 106 g
ii) Mass of 0.15 mole = 0.15 ×106 = 15.9 g
iii) 106 g = 1 mole
1 g = 1/106 mole
3.5 g = 3.5 ×1/106 = 0.033 mole

Q2.15

i) 0.018 mol is equivalent to 2.2 g of benzoic acid
so 1 mol is equivalent to 2.2/0.018 g = 122.2 g

ii) *molar mass* of benzoic acid = 122.2 g mol^{-1}
RMM = 122.2

Q2.16

i) 1000 mL = 1L
1 mL = 0.001 L
10 mL = 0.01 L

ii) 1 μL = 1 × 10^{-6} L
11.6 μL = 11.6 × 10^{-6} L = 1.16 × 10^{-5} L

iii) 1 L = 1000 mL
0.067 L = 0.067 × 1000 = 67 mL

iv) 1 L = 1 × 10^{6} μL
2.6×10^{-7} L = 2.6×10^{-7} × 10^{6} μL = 0.26 μL

Q2.17

Use equation [2.7]
Number of moles = 0.02 mol
Volume = 100mL = 0.1 L
Concentration = 0.02 / 0.1 = 0.2 mol L^{-1} (or 0.2 mol dm^{-3})

Q2.18

Use equation [2.7]
1 mol of NaOH has a mass of 40.0 g
1 g of NaOH is equivalent to 1/40.0 mol
5.6 g of NaOH is equivalent to 5.6/40.0 moles = 0.14 mol
75 mL = 0.075 L
Concentration = Number of moles / Volume, so
Concentration = 0.14 mol / 0.075 L = 1.867 mol L^{-1} (or 1.867 mol dm^{-3})

Q2.19

a concentration of 1 mol L^{-1} has 58.4 g in 1 L
a concentration of 0.1 mol L^{-}1 has 0.1 × 58.4 g = 5.84 g in 1 L
a concentration of 0.1 mol L^{-}1 has 0.1 × 58.4 g = 5.84 g in 1000 mL
1000 mL needs 5.84 g
So 50 mL needs 5.84 × 50/1000 = 0.292 g

Q2.20

Use equation [2.7]
For a concentration of 0.5 M (0.5 mol L^{-1}), we must dissolve the equivalent of 0.5 mol to give 1 L of solution.
1 mol of $CuSO_4.5H_2O$ has a mass of 249.7 g
Hence
0.5 mol of $CuSO_4.5H_2O$ has a mass of $249.7 \times 0.5 = 124.85$ g
Hence for a concentration of 0.5 mol L^{-1}
124.85 g should be dissolved to give 1 L of solution.
50 mL = 0.05 L
giving
124.85×0.05 should be dissolved to give 0.05 L of solution.
$124.85 \times 0.05 = 6.2425$ g

Q2.21

In this example the amount of material stays the same but the volume increases - you can use the **dilution equation [2.6]**:
$5 \times 2 = 100 \times C_f$
$C_f = 5 \times 2 / 100 =$ **0.1 mol L^{-1}** (or 0.1 mol dm^{-3})
(The dilution was a 20 fold dilution.)

Q2.22

Use the dilution equation [2.8]:
$V_i \times C_i = V_f \times C_f$
Final volume, $V_f = 100$ mL
Final concentration, $C_f = 0.04$ mol L^{-1} (40mM = 40/1000 M = 0.04 mol L^{-1})
Initial concentration, $C_i = 0.8$ mol L^{-1}
Hence the only term left to find is the initial volume, V_i, that was put into the 100 mL flask.
Rearranging the dilution equation:
$V_i = V_f \times C_f / C_i = 100 \times 0.04 / 0.8 = 5$ mL

Q2.23

Use the dilution equation [2.8]:
$V_i \times C_i = V_f \times C_f$
Final volume, $V_f = 100$ mL
Final concentration, $C_f = 30$mM
Initial concentration, $C_i = 0.15$ M = 150 mM   changing to same units as C_f
Hence the only term left to find is the initial volume, V_i, that was put into the 100 mL flask.
Rearranging the dilution equation:
$V_i = V_f \times C_f / C_i = 100 \times 30 / 150 = 20$ mL

Q2.24

Use equations [2.13] & [2.14]

i) 360 degrees is one complete rotation.
2π radians is one complete rotation
360 degrees = 2π radians

ii) 90 degrees = $90 \times \pi/180$ radians = $\pi/2$ radians

iii) 170 degrees = $170 \times \pi/180$ radians = 2.967 radians

iv) 1.0 radian = $180/\pi$ degrees = 57.30 degrees

v) 2.1 radians = $2.1 \times 180/\pi$ degrees = 120.32 degrees

vi) 3.5π radians = $3.5\pi \times 180/\pi$ degrees = 3.5×180 degrees = 630 degrees

Q2.25

The distance raised is the arc length of a 40^{o} movement.
$40^{o} = 40 \times \pi/180$ radians = 0.698 radians
The radius is 10cm, so the arc length s = 10cm $\times$ 0.698 = 6.98 cm

Q2.26

Use equation [2.15]
In a right-angled triangle, the height of the tree forms the Opposite side from the angle of 30^{o}, and the length of the shadow is the Adjacent side, $A = 15$.
We use the tan function: $\tan\theta = O/A$, giving
$O/15 = \tan(30^{o}) = 0.577$
Hence the length of the opposite side, $O = 15 \times 0.577 = 8.66$m

Q2.27

Use Pythagoras - equation [2.16].

One side, $O = 100$, and the hypotenuse, $H = 180$, giving:
$180^2 = 100^2 + A^2$ where A is the other side.

This gives:
$32400 = 10000 + A^2$

$A^2 = 22400$ and so A =149.7m

Q2.28

Use equation [2.17]

θ^{o} (degrees)	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	θ (radians)
20	0.3420	0.9397	0.3640	0.3491

10	0.1736	0.9848	0.1763	0.1745
5	0.0872	0.9962	0.0875	0.0873
1	0.0175	0.9998	0.0175	0.0175
0	0	1	0	0

Note that for very small angles [2.17]:
$\sin(\theta) \approx \tan(\theta) \approx \theta$ radians
$\cos(\theta) \approx 1.0$

Q2.29

If 1 rad = 57°, then 0.57° = 1/100 rad.
As the angle is small, we can assume, [2.10], that θ (in radians) = Diameter/Distance
Hence,
1/100 = 0.01 = Diameter/384000
Diameter = 0.01 × 384000 = 3840 km.

Q2.30

The smallest angle will be between the sides with lengths 4 and 5 respectively.
The opposite side, *O*, will have length 3.
The hypotenuse, *H*, (the side opposite the right-angle) will have length 5.
The adjacent side, *A*, will have length 4.
If '*x*' is the value of the smallest angle, using [2.15]:

i) $\sin(x) = O/H = 3/5 = 0.6$
$x = \sin^{-1}(0.6) = 36.87^{\circ}$ or 0.6435 radians

ii) $\cos(x) = A/H = 4/5 = 0.8$
$x = \cos^{-1}(0.8) = 36.87^{\circ}$ or 0.6435 radians

iii) $\tan(x) = O/A = 3/4 = 0.75$
$x = \tan^{-1}(0.75) = 36.87^{\circ}$ or 0.6435 radians

Q2.31

= D1 * TAN(D3*PI()/180) *or* =D1 * TAN(RADIANS(D3))