Powers

(see Bookmarks for index)

Powers and Indices

The expression **a**^{2} is pronounced **'a squared'** or **'a to the power of 2'**.
The '2' is the **index or power,** and 'a' is the **base**.
The plural of index is indices.

A **positive** power shows multiplication: $p^4 = p \times p \times p \times p$

A **negative** power shows division

$$p^{-4} = \frac{1}{(p \times p \times p \times p)}$$

$$p^{-4} = \frac{1}{p^4}$$

Any number to the power of ‘0’ is equal to 1:- $p^0 = 1$

Any number to the power of ‘1’ is itself:- $p^1 = p$

The Reciprocal of p is p raised to the **power of ‘-1’** $= \frac{1}{p} = p^{-1}$

Multiplying and Dividing Powers

When **multiplying** (**dividing**) *numbers which have the same **'base'***, the powers simply **ADD (SUBTRACT)**.

$$p^m \times p^n = p^{(m+n)} \qquad 4^2 \times 4^3 = 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 4^{2+3}$$

$$\frac{p^m}{p^n} = p^{(m-n)} \qquad \frac{6^5}{6^3} = \frac{6 \times 6 \times \cancel{6} \times \cancel{6} \times \cancel{6}}{\cancel{6} \times \cancel{6} \times \cancel{6}} = 6 \times 6 = 6^2 = 6^{5-3}$$

For example, using ‘10’ as the base:

$$10^2 \times 10^5 = 10^{2+5} = 10^7$$

$$\frac{10^{13}}{10^7} = 10^{13-7} = 10^6$$

Power of a Power

Raising a **power to another power** multiplies the powers:

$$(p^m)^n = p^{m \times n}$$

For example (with 4 as the base)

$(4^3)^2 = (4\times4\times4) \times (4\times4\times4) = 4^6 = 4^{3\times2}$

Square Roots

The **square root** of a number **squared** gives the **original number**:

$(\sqrt{9})^2 = 9$

$\sqrt{9} \times \sqrt{9} = 9$

Square root of '9' could be +3 **OR** -3:

$3 \times 3 = 9$

$(-3) \times (-3) = 9$

A number has both a **Positive root** and a **Negative root**.

A **negative number** does not have a 'real' square root.

Taking the square root of a number is the **inverse operation** to raising to a power:

$7^2 = 49$	'49 is the square of 7' **which gives**
$\sqrt{49} = \pm 7$	'7 (plus or minus) is the square root of 49'

Roots & Fractional Powers

Roots appear when we have **fractional** powers.

A **power of ½** is equivalent to the **square root** of the number.

$p^{1/2} = \sqrt{p}$

We can check that this is true:

$(\sqrt{p})^2 = (p^{1/2})^2 = p^{2\times 1/2} = p^1 = p$

Similarly for the **cube root**, $\sqrt[3]{p} = p^{1/3}$:

$(\sqrt[3]{p})^3 = p$

$(\sqrt[3]{p})^3 = (p^{1/3})^3 = p^{3\times 1/3} = p^1 = p$

$2^3 = 8$	'8 is the cube (power of 3) of 2' **which gives**
$\sqrt[3]{8} = 2$	'2 is the cube root of 8'