

## Fractions

(see Bookmarks for index)

### Fraction as a Ratio

A fraction is a **ratio** between two numbers - the **numerator** (on top) and the **denominator** (underneath):

For example the **ratio** 8:13 can be written as the **fraction**:

$$\frac{8}{13}$$

'8' is the **numerator** and '13' is the **denominator**.

NB Any number divided by '1' equals the number itself:

$$\frac{6}{1} = 6$$

### Keeping the Ratio

The fraction keeps the **same ratio** if you multiply (or divide) **both** the numerator and the denominator by the **same** number, e.g.

Multiplying top and bottom by '4':

$$\frac{2}{5} \Rightarrow \frac{2 \times 4}{5 \times 4} \Rightarrow \frac{8}{20}$$

Dividing top and bottom by '3':

$$\frac{18}{15} \Rightarrow \frac{18 \div 3}{15 \div 3} \Rightarrow \frac{6}{5}$$

### Reciprocal

The **reciprocal** of a number (e.g. 'x') is '1' divided by that number:

$$= \frac{1}{x}$$

For example, the reciprocal of 4  $\rightarrow \frac{1}{4} \rightarrow 0.25$

The reciprocal of a **fraction** turns the fraction 'upside down', e.g.

$$\text{Reciprocal of } \frac{3}{4} = \frac{1}{\frac{3}{4}} \Rightarrow \frac{4}{3}$$

## Simplifying & Cancelling

It is often possible to **simplify** a fraction by dividing both top and bottom by the **same number** - a process called **cancelling**, e.g.

The fraction  $84/112$  simplifies to  $3/4$  by dividing top and bottom by '4' and then dividing by '7':

$$\frac{84}{112} \Rightarrow \frac{84/\cancel{4}}{112/\cancel{4}} \Rightarrow \frac{21}{28} \Rightarrow \frac{21/\cancel{7}}{28/\cancel{7}} \Rightarrow \frac{3}{4}$$

In cancelling, **remember** to divide the **whole** of the numerator and/or denominator, e.g.

$$\frac{9+6x}{3} \Rightarrow \frac{(9+6x)/\cancel{3}}{\cancel{3}/3} \Rightarrow \frac{3+2x}{1} \Rightarrow 3+2x$$

## Lowest Common Denominator

A **common denominator** for several fractions is a number that is a **simple multiple** of each of the individual denominators.

For example a common denominator for the following fractions:

$$\frac{2}{3}, \quad \frac{1}{2}, \quad \frac{3}{5}$$

would be '**30**' because the denominators 3, 2 and 5 are all simple factors of 30, i.e. they all divide into 30 in simple whole numbers (10, 15 and 6):

$$30 = 3 \times 10$$

$$30 = 2 \times 15$$

$$30 = 5 \times 6$$

We can transform each of the above fractions to get a common denominator of '30', by multiplying both top and bottom of each fraction by the multiples (10, 15 and 6) as above:

$$\frac{2}{3} \Rightarrow \frac{2 \times 10}{3 \times 10} \Rightarrow \frac{20}{30} \quad \text{Multiplying top and bottom by 10}$$

$$\frac{1}{2} \Rightarrow \frac{1 \times 15}{2 \times 15} \Rightarrow \frac{15}{30} \quad \text{Multiplying top and bottom by 15}$$

$$\frac{3}{5} \Rightarrow \frac{3 \times 6}{5 \times 6} \Rightarrow \frac{18}{30} \quad \text{Multiplying top and bottom by 6}$$

We use common denominators when adding or subtracting fractions.

It is **always** possible to find a common denominator by **multiplying all the denominators** together. For example, in the above example:

$$3 \times 2 \times 5 = 30$$

Often it is possible (but not necessary) to get a **lower** common denominator than that obtained by multiplying all the denominators.

For example the following fractions:

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{4}$$

have a common denominator =  $2 \times 6 \times 4 = 48$ , but by simple inspection of the numbers we can see that '12' would be also be common denominator:

$$12 = 2 \times 6$$

$$12 = 6 \times 2$$

$$12 = 4 \times 3$$

## Adding and Subtracting

Before **adding or subtracting** fractions, they must be transformed so that they all have a **common denominator** as above.

For example, to add  $1/2$  and  $1/4$  we first find that '4' is a common denominator and transform  $1/2$  into  $2/4$  (= two quarters). We then add two quarters to one quarter, giving three quarters:

$$\frac{1}{2} + \frac{1}{4} \Rightarrow \frac{2}{4} + \frac{1}{4} \Rightarrow \frac{2+1}{4} \Rightarrow \frac{3}{4}$$

In the more complicated example below, the common denominator is '30', so we first convert  $2/3$  to  $20/30$ ,  $1/2$  to  $15/30$  and  $3/5$  to  $18/30$ :

$$\begin{aligned} \frac{2}{3} - \frac{1}{2} + \frac{3}{5} &\Rightarrow \frac{2 \times 10}{3 \times 10} - \frac{1 \times 15}{2 \times 15} + \frac{3 \times 6}{5 \times 6} \\ &\Rightarrow \frac{20}{30} - \frac{15}{30} + \frac{18}{30} \Rightarrow \frac{20 - 15 + 18}{30} \Rightarrow \frac{23}{30} \end{aligned}$$

## Multiplying and Dividing

When **multiplying** fractions, the numerators and denominators can be multiplied separately:

$$\frac{2}{5} \times \frac{4}{3} \Rightarrow \frac{2 \times 4}{5 \times 3} \Rightarrow \frac{8}{15}$$

**Dividing** by a fraction is the same as **multiplying by the fraction turned upside down** (i.e its reciprocal):

$$\frac{2/5}{4/3} \Rightarrow \frac{2}{5} \times \frac{3}{4} \Rightarrow \frac{2 \times 3}{5 \times 4} \Rightarrow \frac{6}{20} \Rightarrow \frac{3}{10}$$

## Improper Fractions

**Converting an 'improper' fraction** to a 'mixed' number - divide the numerator by the denominator and note the remainder.

For  $35/8$ , we find that 35 divided by 8 is 4 with a remainder of 3.

$$\frac{35}{8} = 4\frac{3}{8}$$

**Converting 'mixed' numbers** to fractions - convert the **whole number** to an improper fraction using the same denominator as for the fractional part of the number, and then add to the fractional part:

$$4\frac{3}{8} \Rightarrow 4 + \frac{3}{8} \Rightarrow \frac{32}{8} + \frac{3}{8} \Rightarrow \frac{32+3}{8} \Rightarrow \frac{35}{8}$$

## Fractions and Decimals

Turning a **fraction into a decimal** - use your calculator to divide the numerator by the denominator:

$$\frac{3}{8} \Rightarrow 3 \div 8 \Rightarrow 0.375$$

Turning a **decimal into a fraction** - put the decimal as a fraction with a denominator of '1' and then multiply top **and** bottom by whatever factor of 10 (eg 100, 1000, etc) will clear the decimal point to the extreme right.

$$0.375 \Rightarrow \frac{0.375}{1} \Rightarrow \frac{0.375 \times 1000}{1 \times 1000} \Rightarrow \frac{375}{1000}$$

The result can then be simplified by cancelling (by 5 and then by 25)

$$\frac{375}{1000} \Rightarrow \frac{75}{200} \Rightarrow \frac{3}{8}$$