Statistical Tables
&
Formula Sheet

compiled by

Dr Graham Currell

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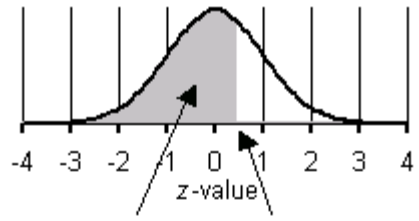
Contents: Page

Cumulative *z*-Areas for Standard Normal Distribution

Area shown is the cumulative probability between z = - Infinity and the given value of *z*

Initial example for *z* = 0.445
gives Probability Area = 0.6718



0.6718 $z =$ **0.445**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Critical Values:- *t* - Statistic & Chi-Squared, χ^2

		t - Statistic		Chi-Squared	
Tails	**Degrees**	**Significance**		**Significance**	
	of freedom	**95%**	**99%**	**95%**	**99%**
1 or 2	***df***	**0.05**	**0.01**	**0.05**	**0.01**
2	1	12.71	63.66		
1		6.31	31.82	3.84	6.63
2	2	4.30	9.92		
1		2.92	6.96	5.99	9.21
2	3	3.18	5.84		
1		2.35	4.54	7.81	11.34
2	4	2.78	4.60		
1		2.13	3.75	9.49	13.28
2	5	2.57	4.03		
1		2.02	3.36	11.07	15.09
2	6	2.45	3.71		
1		1.94	3.14	12.59	16.81
2	7	2.36	3.50		
1		1.89	3.00	14.07	18.48
2	8	2.31	3.36		
1		1.86	2.90	15.51	20.09
2	9	2.26	3.25		
1		1.83	2.82	16.92	21.67
2	10	2.23	3.17		
1		1.81	2.76	18.31	23.21
2	11	2.20	3.11		
1		1.80	2.72	19.68	24.73
2	12	2.18	3.05		
1		1.78	2.68	21.03	26.22
2	13	2.16	3.01		
1		1.77	2.65	22.36	27.69
2	14	2.14	2.98		
1		1.76	2.62	23.68	29.14
2	15	2.13	2.95		
1		1.75	2.60	25.00	30.58
2	16	2.12	2.92		
1		1.75	2.58	26.30	32.00
2	17	2.11	2.90		
1		1.74	2.57	27.59	33.41
2	18	2.10	2.88		
1		1.73	2.55	28.87	34.81
2	19	2.09	2.86		
1		1.73	2.54	30.14	36.19
2	20	2.09	2.85		
1		1.72	2.53	31.41	37.57
2	25	2.06	2.79		
1		1.71	2.49	37.65	44.31
2	50	2.01	2.68		
1		1.68	2.40	67.50	76.15
2	Infinity	1.96	2.58		
1		1.64	2.33		

Critical *F* Values at 0.05 (95%) Significance

		df_N = Degrees of freedom for Numerator														
		df_D = Degrees of freedom for Denominator														
Tails	df_N =	1	2	3	4	5	6	7	8	9	10	15	20	30	50	100
1 or 2	df_D															
2	1	648	799	864	900	922	937	948	957	963	969	985	993	1001	1008	1013
1		161	199	216	225	230	234	237	239	241	242	246	248	250	252	253
2	2	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5
1		18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
2	3	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.3	14.2	14.1	14.0	14.0
1		10	10	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.62	8.58	8.55
2	4	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.66	8.56	8.46	8.38	8.32
1		7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.75	5.70	5.66
2	5	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.43	6.33	6.23	6.14	6.08
1		6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.50	4.44	4.41
2	6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.27	5.17	5.07	4.98	4.92
1		5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.81	3.75	3.71
2	7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.57	4.47	4.36	4.28	4.21
1		5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.38	3.32	3.27
2	8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.10	4.00	3.89	3.81	3.74
1		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.08	3.02	2.97
2	9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.77	3.67	3.56	3.47	3.40
1		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.86	2.80	2.76
2	10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.52	3.42	3.31	3.22	3.15
1		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.70	2.64	2.59
2	15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.86	2.76	2.64	2.55	2.47
1		4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.25	2.18	2.12
2	20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.57	2.46	2.35	2.25	2.17
1		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.04	1.97	1.91
2	30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.31	2.20	2.07	1.97	1.88
1		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.84	1.76	1.70
2	50	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38	2.32	2.11	1.99	1.87	1.75	1.66
1		4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.87	1.78	1.69	1.60	1.52
2	100	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24	2.18	1.97	1.85	1.71	1.59	1.48
1		3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.77	1.68	1.57	1.48	1.39

Example:

Critical *F*-value for 1-tailed test with $df_N = 15$ and $df_D = 8$ at 0.05 significance:

$F_{1,0.05,15,8} = 3.22$

Critical Values at 0.05 (95%) Significance for

- **Pearson's correlation coefficient, *r***
- **Spearman's Rank correlation coefficient, r_S**
- **Wilcoxon lower limit, W_L**

Tails	Sample size	Pearson's	Spearman's	Wilcoxon
1 or 2	*n*	*r*	r_s	W_L
2	4	0.950		
1		0.900		
2	5	0.878	-	
1		0.805	0.900	
2	6	0.811	0.886	-
1		0.729	0.829	2
2	7	0.754	0.786	2
1		0.669	0.714	3
2	8	0.707	0.738	3
1		0.622	0.643	5
2	9	0.666	0.683	5
1		0.582	0.600	8
2	10	0.632	0.648	8
1		0.549	0.564	10
2	11	0.602	0.623	10
1		0.521	0.523	13
2	12	0.576	0.591	13
1		0.497	0.497	17
2	13	0.553	0.566	17
1		0.476	0.475	21
2	14	0.532	0.545	21
1		0.456	0.457	25
2	15	0.514	0.525	25
1		0.441	0.441	30
2	16	0.497	0.507	29
1		0.426	0.425	35
2	17	0.482	0.490	34
1		0.412	0.412	41
2	18	0.468	0.476	40
1		0.400	0.399	47
2	19	0.456	0.462	46
1		0.389	0.388	53
2	20	0.444	0.450	52
1		0.378	0.377	60
2	25	0.396	0.400	89
1		0.337	0.336	100
2	30	0.361	0.364	137
1		0.306	0.305	151

Mann-Whitney Lower Limit, U_L, at 0.05 (95%) Significance

Tails	Sample sizes																					
1 or 2	n_2	n_1 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2											0	1	1	1	1	1	2	2	2	2
1								0	1	1	1	1	2	2	3	3	3	3	4	4	4
2	3					0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
1					0	1	2	2	3	4	4	5	5	6	7	7	8	9	9	10	11
2	4				0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14
1				0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
2	5			0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
1				1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
2	6			1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
1				2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
2	7			1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
1			0	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37	39
2	8		0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
1			1	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44	47
2	9		0	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
1			1	4	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
2	10		0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
1			1	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58	62
2	11		0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
1			1	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65	69
2	12		1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
1			2	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72	77
2	13		1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
1			2	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80	84
2	14		1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	69	74	78	83
1			3	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87	92
2	15		1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
1			3	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94	100
2	16		1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
1			3	8	14	19	25	30	36	42	48	54	60	65	71	77	83	89	95	101	107
2	17		2	6	11	17	22	28	34	39	45	51	57	63	69	75	81	87	93	99	105
1			3	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96	102	109	115
2	18		2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
1			4	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109	116	123
2	19		2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
1			4	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	130
2	20		2	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127
1			4	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138

Formula Sheet

Propagating Uncertainty

$$u(r) = \sqrt{u(r)^2} = \sqrt{u(a)^2 + u(b)^2} \qquad Ru(r) = \sqrt{Ru(r)^2} = \sqrt{Ru(a)^2 + Ru(b)^2}$$

F-test

$$F_{STAT} = \frac{s_A^2}{s_B^2} \quad \left(= \frac{MS(B)}{MS(W)} \text{ in ANOVA} \right) \qquad dfA = n_A - 1 \text{ and } dfB = n_B - 1$$

CI of the Population Mean

$$CI(\mu, X\%) = \bar{x} \pm \left\{ t_{2,\alpha,(n-1)} \times \frac{s}{\sqrt{n}} \right\} \qquad df = n - 1$$

One-sample *t*-test

$$t_{STAT} = \frac{(\bar{x} - \mu_O)}{s / \sqrt{n}} \qquad df = n - 1$$

Two-sample *t*-test

$$t_{STAT} = \frac{(\bar{x}_A - \bar{x}_B)}{s' \times \sqrt{\{1/n_A + 1/n_B\}}} \qquad s' = \sqrt{\frac{\{(n_A - 1)s_A^2 + (n_B - 1)s_B^2\}}{(n_A + n_B - 2)}}$$

$$df = n_A + n_B - 2$$

CI of the proportion

$$CI(\Pi, 95\%) = P \pm 1.96 \times \sqrt{\frac{P \times (1 - P)}{n}}$$

One-proportion test

$$z = \frac{P - \Pi_O}{\sigma_P} = \frac{P - \Pi_O}{\sqrt{\frac{P \times (1 - P)}{n}}}$$

Two-proportion test

$$z = \frac{P_A - P_B}{\sqrt{P'(1 - P') \times (1/n_A + 1/n_B)}} \qquad P' = \frac{n_A P_A + n_B P_B}{n_A + n_B}$$

Analysis of Variances

$$SS_x = \sum_i (x_i - \bar{x})^2 \qquad MS_x = \frac{SS_x}{df}$$

$MS(B) = n \times VSM = (n \times \sigma_F^2) + \sigma_E^2$ $\qquad df(B) = k - 1$

$MS(W) = MSV = \sigma_E^2$ $\qquad df(W) = N - k = k(n - 1)$

Tukey: $q_{STAT} = \frac{(\bar{x}_{TOP} - \bar{x}_{BOTTOM})}{\sqrt{\frac{MS_{ERROR}}{2}\left(\frac{1}{n_{TOP}} + \frac{1}{n_{BOTTOM}}\right)}}$ $\qquad q_{STAT} = \sqrt{2} \times$ MINITAB 'T-value'

Regression

$$r^2 = \frac{m^2 \times SS_x}{SS_y} = \frac{SS_y - SS_{RESID}}{SS_y} = 1 - \frac{SS_{RESID}}{SS_y} \qquad SE_{xy} = \sqrt{\frac{SS_{RESID}}{n - 2}}$$

$$u(x_O) = \frac{SE_{xy}}{m} \times \sqrt{\left\{ \frac{1}{k} + \frac{1}{n} + \frac{(y_O - \bar{y})^2}{m^2 \times \sum_i (x_i - \bar{x})^2} \right\}} \qquad x_O = \frac{y_O - c}{m}$$

Non-Parametric Tests

Wilcoxon

$W(+) + W(-) = 0.5 \times n \times (n + 1)$

Mann-Whitney

$U_x = W_x - n_x(n_x + 1)/2$