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# 8.3a Deriving Standard Uncertainty

Before it is possible to 'combine' uncertainties (as in 8.3.3), it is necessary to get them in the same form, preferably as 'standard uncertainties'. However, uncertainties are often described in a variety of different forms, and the conversion required for some common forms is given below:

### 8.3a.1 Confidence Interval

When the uncertainty is given as a 95% Confidence Interval, then the Confidence Deviation, *Cd*, should be divided by 1.96 (for 99% confidence divide by 2.58).
e.g.

$$u(x) = \frac{Cd(\mu, 95\%)}{1.96}$$

### 8.3a.2 Standard Deviation of Small Samples

If the uncertainty is given as the standard deviation, *s*, of a small sample (*n* is small), then, to be consistent with other uncertainties, we recommend that the uncertainty is first converted into a Confidence Interval and then treated as above. Effectively this would modify equation [8.8] to give:

$$u(x) = \frac{t_{2,\alpha,n-1}}{1.96} \times \frac{s}{\sqrt{n}}$$

### 8.3a.3 Tolerance or Rectangular (or Limiting) Uncertainty

Uncertainty is often given as a *limiting* uncertainty, or *tolerance*, that defines the *absolute* range of possible values. For example, a class B volumetric flask may have a volume certified as 25 mL ± 0.08 mL, which implies that the volume may lie anywhere within the range 24.92 to 25.08 but will *not* lie outside this range. In effect this is a *rectangular* distribution.

When combining a limited (or rectangular) uncertainty range $V \pm \Delta$ with other uncertainties, it is necessary to take into account the fact that the statistical probabilities for the *combined* value will have a *non-rectangular* distribution. To accommodate these probabilities, it is first necessary to calculate an effective *standard uncertainty* which is equivalent to the rectangular distribution. This is achieved by dividing the tolerance by √3,
i.e. $u_V = \Delta / \sqrt{3}$.
This is approximately equal to multiplying by 0.6,
i.e. $u_V \approx 0.6 \times \Delta$.

**Example**

A 100 mL grade A pipette has a limiting uncertainty of ± 0.08 mL.
A technician introduces a standard deviation uncertainty, $u(V)$ = 0.09 mL, when setting the fluid level to the calibrated mark.

The technician uses the pipette to deliver a volume of fluid, $V$ =100 mL.

i) Calculate the standard uncertainty, $u(V)$, in the volume.

ii) State the volume of fluid with an appropriate expression of uncertainty.

Answer

i) Uncertainty in the volume is an additive combination of the uncertainty in the calibration mark and the uncertainty in the setting of the level.

The uncertainty in the calibration mark is given by the limiting uncertainty ± 0.08 mL, which is converted to a *standard uncertainty* by dividing by √3:

$u(V_{CAL}) = 0.08 / \sqrt{3} = 0.046$

The *standard uncertainty* in setting the fluid level, $u(V_{SET})$ = 0.09 mL

The combined *standard uncertainty* is given by:

$$u(V) = \sqrt{u(V_{CAL})^2 + u(V_{SET})^2} = \sqrt{0.046^2 + 0.09^2} = 0.10$$

ii) Using a coverage factor of 2 (for a confidence interval of approximately 95%):

Final volume, $V$ = 100.00 ±0.20 mL