Essential Mathematics & Statistics for Science, 2nd Ed
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(*Wiley-Blackwell, 2009*)

Revision Mathematics

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Contents:

0.1 Number Line & Inequalities

Numbers, Number Line, Multiplication, Modulus, Inequalities

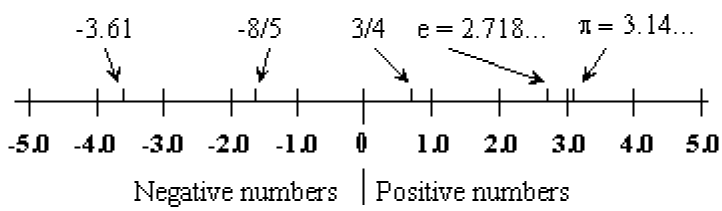
0.1.1 Numbers

Numbers can be expressed in various forms -

Integers	e.g. 3, -4, 146
Decimal form	e.g. 0.045, 156.98
Fractions	e.g. -7/8, 23/9
Scientific (Standard) Notation (see 2.1.2)	e.g. 2.3×10^3

0.1.2 Number Line

Using the 'Number Line', helps you sort out problems that involve adding and subtracting positive and negative numbers.



Some specific numbers have special significance in mathematics and science, e.g.

e (Euler's number) = 2.718... and π (pi) = 3.14....

Subtraction, '-', means 'move' to the **left** in the above line - towards more negative values.
Addition, '+', means 'move' to the **right** in the above line - towards more positive values.

The positive or negative sign in front of a single number shows the **direction** of that 'number' away from '0'.

For example:

-2 (= 0 - 2) means move a distance '2' to the **left** starting from '0'
4 - 6 (= -2) means move a distance '6' to the **left** starting from '+4', ending at '-2'

0.1.3 Multiplication and Division

a **plus** times (or divided by) a **plus** gives a **plus:-**	e.g. $+6 \times (+2) = +12$
a **plus** times (or divided by) a **minus** gives a **minus:-**	e.g. $+6 / (-2) = -3$
a **minus** times (or divided by) a **plus** gives a **minus**:-	e.g. $-6 \times (+2) = -12$
a **minus** times (or divided by) a **minus** gives a **plus**:-	e.g. $-6 / (-2) = +3$

Check that you understand each of the following examples:

$-(-2) = -1 \times (-2) = +2$

$4 - (-4) = 4 + 4 = 8$

$1 / (-3) = -(1/3)$

$3/5 - 2/5 = (3 - 2)/5 = 1/5$

$-4(2 - 3) = -4 \times (2 - 3) = -8 + 12 = +4$

$-4(2 - 3) = -4 \times (-1) = +4$

$-(a - b) = -a - (-b) = -a + b = (b - a)$

$-(X - Y) = +(Y - X)$

0.1.4 Modulus

The **Modulus** of a number, x, is written as |x|, and is the magnitude of a number WITHOUT the sign,
For example:

Modulus of -2 = **|-2|** = 2
Modulus of +2 = **|+2|** = 2

0.1.5 Inequalities

a > b means that 'a' is more **positive** than 'b', and
a < b means that 'a' is more **negative** than 'b'.

On the number line where **positive** numbers are to the **right**:
a > b means that 'a' is to the **right** of 'b', and
a < b means that 'a' is to the **left** of 'b'.

Note also that, if $a > b$ then $1/a < 1/b$

0.2 BODMAS

0.2.1 Brackets, Power Of, Divide, Multiply, Add, Subtract

BODMAS (or BIDMAS, BEDMAS) gives **the order** in which you should carry out calculations:

Brackets (parentheses) first - then the
Powers **O**f**, I**ndices, **E**xponents- then any
Divisions or
Multiplications - then any
Adding or
Subtracting

Examples - check that you understand **all** of the following equations:

$3 + 2 \times 4 = 11$
$(3 + 2) \times 4 = 20$
$4 \times (3 + 2) = 20$
$4 \times 3 + 2 = 14$
$6 - 4 \div 2 = 4$
$(6 - 4) \div 2 = 1$
$3 \times 2^2 = 3 \times 4 = 12$
$(3 \times 2)^2 = 6^2 = 36$

0.2.2 BODMAS in Excel

EXCEL spreadsheet uses BODMAS rules when writing out equations.
(see Tutorial **Skills** for use of EXCEL)

For example, the equation '= E3+D2*F2' will multiply the contents of cells D2 and F2, and **then** add the result to the contents of E3.

If you wish to add E3 to D2 **before** multiplying by F2 you should write '= (E3+D2)*F2'

The equation = (E3 + D2)^F2 will add E3 to D2 and **then** raise the sum to the power of F2.

The equation = E3 + D2^2 will square the contents of D2 **before** adding to E3.

0.3 Fractions

Fractions, Keeping the Ratio, Reciprocal, Simplifying & Cancelling,
Lowest Common Denominator' Adding & Subtracting, Multiplying & Dividing,
Improper Fractions, Fractions & Decimals

0.3.1 Fractions

A fraction is a **ratio** between two numbers -
the **numerator** (on top) and the **denominator** (underneath):

For example the **ratio** 8:13 can be written as the **fraction**:

$$\frac{8}{13}$$

'8' is the **numerator** and '13' is the **denominator**.

NB Any number divided by '1' equals the number itself:

$$\frac{6}{1} = 6$$

0.3.2 Keeping the Ratio

The fraction keeps the **same ratio** if you multiply (or divide) **both** the numerator and the denominator by the **same** number, e.g.

Multiplying top and bottom by '4':

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

Dividing top and bottom by '3':

$$\frac{18}{15} = \frac{18/3}{15/3} = \frac{6}{5}$$

0.3.3 Reciprocal

The **reciprocal** of a number (e.g. 'x') is '1' divided by that number:

$$= \frac{1}{x}$$

The reciprocal of a **fraction** turns the fraction 'upside down', e.g.

$$\text{(Reciprocal of 3/4)} = \frac{1}{3/4} = \frac{4}{3}$$

0.3.4 Simplifying & Cancelling

It is often possible to **simplify** a fraction by dividing both top and bottom by the **same number** - a process called **cancelling**, e.g.

The fraction 84/112 simplifies to 3/4 by dividing top and bottom by '4' and then dividing by '7':

$$\frac{84}{112} = \frac{84/4}{112/4} = \frac{21}{28} = \frac{21/7}{28/7} = \frac{3}{4}$$

In cancelling, **remember** to divide the **whole** of the numerator and/or denominator, e.g.

$$\frac{9+6x}{3} = \frac{(9+6x)/3}{3/3} = \frac{3+2x}{1} = 3+2x$$

0.3.5 Lowest Common Denominator

A **common denominator** for several fractions is a number that is a **simple multiple** of each of the individual denominators.
For example a common denominator for the following fractions:

$$\frac{2}{3}, \frac{1}{2}, \frac{3}{5}$$

would be **'30'** because the denominators 3, 2 and 5 are all simple factors of 30, i.e. they all divide into 30 in simple whole numbers (10, 15 and 6):

$30 = 3 \times 10$
$30 = 2 \times 15$
$30 = 5 \times 6$

We can transform each of the above fractions to get a common denominator of '30', by multiplying both top and bottom of each fraction by the multiples (10, 15 and 6) as above:

$$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$$

$$\frac{1}{2} = \frac{1 \times 15}{2 \times 15} = \frac{15}{30}$$

$$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$$

We use common denominators when adding or subtracting fractions.

It is **always** possible to find a common denominator by **multiplying all the denominators** together. For example, in the above example:
$3 \times 2 \times 5 = 30$

Often it is possible (but not necessary) to get a **lower** common denominator than that obtained by multiplying all the denominators.
For example the following fractions:

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{4}$$

have a common denominator = $2 \times 6 \times 4 = 48$, but by simple inspection of the numbers we can see that '12' would be also be common denominator:

$12 = 2 \times 6$
$12 = 6 \times 2$
$12 = 4 \times 3$

For more analytical ways of finding the lowest common denominator you should consult other basic mathematics books.

0.3.6 Adding and Subtracting

Before **adding or subtracting** fractions, they must be transformed so that they all have a **common denominator**. (See common denominator).
For example, to add 1/2 and 1/4 we first find that '4' is a common denominator and transform 1/2 into 2/4 (= two quarters). We then add two quarters to one quarter, giving three quarters:

$$\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$$

In the more complicated example below, the common denominator is '30', so we first convert 2/3 to 20/30, 1/2 to 15/30 and 3/5 to 18/30:

$$\frac{2}{3}-\frac{1}{2}+\frac{3}{5}=\frac{2\times 10}{3\times 10}-\frac{1\times 15}{2\times 15}+\frac{3\times 6}{5\times 6}=\frac{20}{30}-\frac{15}{30}+\frac{18}{30}$$
$$=\frac{20-15+18}{30}=\frac{23}{30}$$

0.3.7 Multiplying and Dividing

When **multiplying** fractions, the numerators and denominators can be multiplied separately:

$$\frac{2}{5}\times\frac{4}{3}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$$

Dividing by a fraction is the same as **multiplying by its reciprocal** (see reciprocal):

$$\frac{2/5}{4/3}=\frac{2}{5}\div\frac{4}{3}=\frac{2}{5}\times\frac{3}{4}=\frac{2\times 3}{5\times 4}=\frac{6}{20}=\frac{3}{10}$$

0.3.8 Improper Fractions

Converting an 'improper' fraction to a 'mixed' number **-** divide the numerator by the denominator and note the remainder.

For 35/8, we find that 35 divided by 8 is 4 with a remainder of 3.

$$\frac{35}{8} = 4\frac{3}{8}$$

Converting 'mixed' numbers to fractions **-** convert the **whole number** to an improper fraction using the same denominator as for the fractional part of the number, and then add to the fractional part:

$$4\frac{3}{8} = 4 + \frac{3}{8} = \frac{32}{8} + \frac{3}{8} = \frac{32+3}{8} = \frac{35}{8}$$

0.3.9 Fractions and Decimals

Turning a **fraction into a decimal** - use your calculator to divide the numerator by the denominator:

$$\frac{3}{8} = 3 \div 8 = 0.375$$

Turning a **decimal into a fraction -** put the decimal as a fraction with a denominator of '1' and then multiply top **and** bottom by whatever factor of 10 (eg 100, 1000, etc) will clear the decimal point to the extreme right.

$$0.375 = \frac{0.375}{1} = \frac{0.375 \times 1000}{1 \times 1000} = \frac{375}{1000}$$

The result can then be simplified by cancelling (by 5 and then by 25)

$$\frac{375}{1000} = \frac{75}{200} = \frac{3}{8}$$

0.4 Percentages

Percentage, Increase/Decrease

0.4.1 Percentage

A **fraction or ratio** can be expressed as a **percentage** by multiplying the fraction by 100 and dividing out the fraction.

For example: '4' as a percentage of '32' is equal to

$$\frac{4}{32}\times 100=\frac{100}{8}=12.5\%$$

A **percentage** can be expressed as a **fraction** by dividing by 100 and expressing the result as a fraction

For example: 25% expressed as a fraction is 25/100 = 0.25 = 1/4

0.4.2 Percentage Increase or Decrease

In a percentage change calculation we can relate a **New Value** to a **Base Value** and a **Change**:

New = Base + Change

Expressed as **percentages**, the **Base value would be 100%**, giving:

New% = 100 + Chng%

where

$$\text{New\%}=100\times\frac{\text{New}}{\text{Base}}$$

and

$$\text{Chng\%}=100\times\frac{\text{Change}}{\text{Base}}$$

Change and **Chng%** would be **positive** for an **increase** and **negative** for a **decrease**.

The equations can be rearranged to give:

$$\text{New} = \text{Base} \times \left(1 + \frac{\text{Chng\%}}{100}\right)$$

The first step in any problem is to work out which value is the Base Value. This is the value that is equivalent to 100%..
The next step is to use any of the above equations that are useful.

Example:
What is the percentage increase if a person's weight increases from 80 kg to 85 kg?

In this problem, the increase would be expressed as a percentage of the initial value (80 kg).
Hence 80 kg is the Base Value and 85 kg is the New Value.
Then Change = New - Base = 85 - 80 = 5, and
Chng% = 100 × 5/80 = 6.25 %

Example:
The cost of a TV is reduced by 15% to £187. What was the original price, P?

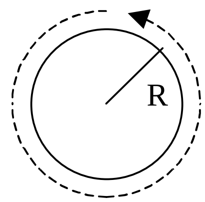
The reduction was 15% of the original price, P. *Hence P is the Base Value*.
The Change is negative because there is a reduction and Chng% = -15, and New = 187.
Hence: 187 = P × (1 - 15/100) = P × (0.85), and then
P = 187 / 0.85 = 220
The original price was £220.

0.5 Areas and Volumes

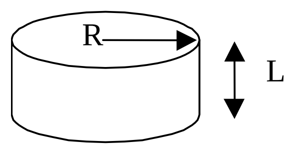
0.5.1 Area Formulae

Area of Circle = $\pi \times (\text{radius})^2$
= $\pi \times R^2$

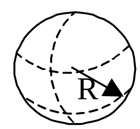
Circumference of a circle = $2\pi R$



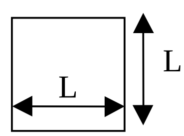
Curved Area of a Cylinder
= Circumference × length
= $2\pi R \times L$



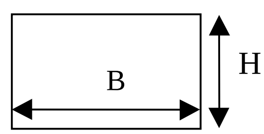
Surface Area of a Sphere (R = radius)
= $4 \times \pi \times R^2$



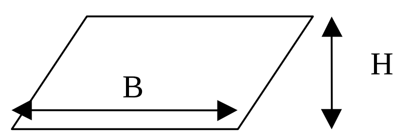
Area of Square = $(\text{length of side})^2$
= L^2



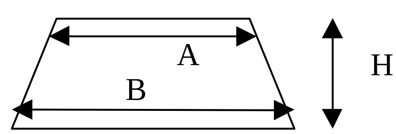
Area of Rectangle = base × height
= $B \times H$



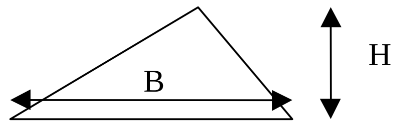
Area of Parallelogram = base × height
= $B \times H$



Area of Trapezium = average length × height
= $(1/2) \times (A + B) \times H$



Area of Triangle = half base × height
= (1/2) × B × H



Example
The main trunk of a tree is 3.6 m high and has an average radius of 0.1 m.
Estimate the area of bark on the trunk.

Surface area of the trunk as a cylinder = $2\pi R \times L = 2\pi \times 0.1 \times 3.6 = 2.26\ \text{m}^2$

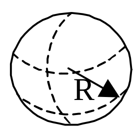
Example
Water covers 71% of the Earth's surface. Estimate the total surface area of water on the Earth.
Radius of the Earth = 6400 km.

Surface area of Earth= $4 \times \pi \times R^2 = 4 \times \pi \times (6400)^2 = 5.15 \times 10^8\ \text{km}^2$
71% of this area = $0.71 \times 5.15 \times 10^8\ \text{km}^2 = 3.65 \times 10^8\ \text{km}^2$

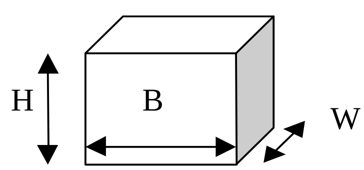
0.5.2 Volume Formulae

Volume of a Sphere (radius = R)

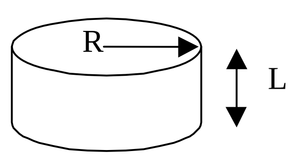
$$= \frac{4}{3} \times \pi \times R^3$$



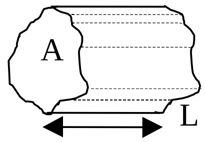
Volume of a Cuboid
= base × width ×height
= $B \times W \times H$



Volume of a Cylinder
= cross-sectional area × length
= $\pi \times R^2 \times L$



Volume of Projected Area
= cross-sectional area × length
= $A \times L$



Example
The main trunk of a tree is 3.6 m high and has an average radius of 0.1 m.
Estimate the volume of the trunk

Volume of the trunk as a cylinder = $\pi \times R^2 \times L = \pi \times 0.1^2 \times 3.6 = 0.11$ m^3

0.5.3 Converting Area and Volume Units

See also 2.2.4 Conversion of Units

Be careful with units of area and volume - it is easy to make simple mistakes.

For example - do not confuse 10 metres square with 10 square metres.
"10 metres square" describes a square area with sides of length 10 m.
The actual area of '10 metres square' equals 10 × 10 = 100 m^2 = 100 square metres!

When converting units it is best to proceed in very small steps - do not try to do it in your head!

Conversion: metres and centimetres

1 m = 100 cm
1 m^2 = 100 × 100 cm^2 = 10000 cm^2 = 1.0×10^4 cm^2
1 m^3 = 100 × 100 × 100 cm^3 = 1000000 cm^3 = 1.0×10^6 cm^3

Hence:

1 cm^2 = 1.0×10^{-4} m^2
1 cm^3 = 1.0×10^{-6} m^3

Conversion: kilometres and metres

1 km = 1000 m = 1.0×10^3 m
1 km^2 = $(1.0 \times 10^3)^2$ m^2 = 1.0×10^6 m^2
1 km^3 = $(1.0 \times 10^3)^3$ m^3 = 1.0×10^9 m^3

Hence:

1 m^2 = 1.0×10^{-6} km^2
1 m^3 = 1.0×10^{-9} km^3

Conversion: mm^2 and m^2

Calculate

$1\ m = 1000\ mm = 1.0 \times 10^3\ mm$

$1\ m^2 = (1.0 \times 10^3)^2\ mm^2 = 1.0 \times 10^6\ mm^2$

$1\ m^3 = (1.0 \times 10^3)^3\ mm^3 = 1.0 \times 10^9\ mm^3$

Hence:

$1\ mm^2 = 1.0 \times 10^{-6}\ m^2$

$1\ mm^3 = 1.0 \times 10^{-9}\ m^3$

Example:

A cuboid measures 2.0 mm by 3.0 mm by 1.0 mm. Calculate the volume

i) in units of mm^3

ii) in units of m^3

i) Volume of a cuboid = $2.0\times3.0\times1.0 = 6.0\ mm^3$

ii) Using conversion from above

$1\ mm^3 = 1.0 \times 10^{-9}\ m^3$

Hence volume = $6.0 \times 1.0 \times 10^{-9}\ m^3 = 6.0 \times 10^{-9}\ m^3$

0.6 Powers, Indices and Roots

Powers & Indices, Multiplying & Dividing Powers, Power of a Power, Square Roots, Roots & Fractional Powers

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0.6.1 Powers and Indices

The expression $\mathbf{a}^2$ is pronounced **'a squared'** or **'a to the power of 2'**.
The '2' is the **index or power,** and 'a' is the **base**.
The plural of index is indices.

Positive power shows multiplication: $p^4 = p \times p \times p \times p$

Negative power shows division

$$p^{-4} = \frac{1}{(p \times p \times p \times p)}$$

$$p^{-4} = \frac{1}{p^4}$$

The **Reciprocal** of p is:-	$p^{-1} = 1/p$
Any number to the power of '0' is equal to 1:-	$p^0 = 1$
Any number to the power of '1' is itself:-	$p^1 = p$

0.6.2 Multiplying and Dividing Powers

When **multiplying** (**dividing**) *numbers which have the same* ***'base'***, the powers simply **ADD (SUBTRACT)**.

$$p^m \times p^n = p^{(m+n)}$$

$$\frac{p^m}{p^n} = p^m \div p^n = p^{(m-n)}$$

$$\frac{p^5}{p^3} = \frac{p \times p \times p \times p \times p}{p \times p \times p} = \frac{p \times p \times \cancel{p} \times \cancel{p} \times \cancel{p}}{\cancel{p} \times \cancel{p} \times \cancel{p}} = \frac{p \times p}{1} = p^2$$

For example (with 10 as the base):

$10^2 \times 10^5 = 10^{2+5} = 10^7$

$10^{13}/10^7 = 10^{13-7} = 10^6$

0.6.3 Power of a Power

Raising a **power to another power**

$(p^m)^n = p^{m \times n}$

For example (with 3 as the base)

$(3^2)^4 = 3^{2 \times 4} = 3^8$

0.6.4 Square Roots

The **square root** of a number **squared** gives the **original number**:

$(\sqrt{9})^2 = 9$

$\sqrt{9} \times \sqrt{9} = 9$

Square root of '9' could be +3 **OR -**3:

$3 \times 3 = 9$

$(-3) \times (-3) = 9$

Note that a number has both a **Positive root** and a **Negative root**.

A **negative number** does not have a 'real' square root.

Taking the square root of a number is the **inverse operation** to raising to a power:

$7^2 = 49$ '49 is the square of 7' **which gives**

$\sqrt{49} = \pm 7$ '7 (plus or minus) is the square root of 49'

0.6.5 Roots & Fractional Powers

Roots appear when we have **fractional** powers.

A **power of ½** is equivalent to the **square root** of the number.

$$p^{1/2} = \sqrt{p}$$

We can check that this is true:

$$(\sqrt{p})^2 = (p^{1/2})^2 = p^{2\times 1/2} = p^1 = p$$

Similarly for the **cube root**, $\sqrt[3]{p} = p^{1/3}$:

$$(\sqrt[3]{p})^3 = p$$

$$(\sqrt[3]{p})^3 = (p^{1/3})^3 = p^{3\times 1/3} = p^1 = p$$

$2^3 = 8$ '8 is the cube of 2' **which gives**

$\sqrt[3]{8} = 2$ '2 is the cube root of 8'